M.Sc. (IT) Final Year

MIT-15

SIMULATION AND MODELLING

मध्यप्रदेश भोज (मुक्त) विश्वविद्यालय – भोपाल

MADHYA PRADESH BHOJ (OPEN) UNIVERSITY - BHOPAL

COURSE WRITERS

Sinchan Banerjee, Former Faculty, Dept. of I.T. and Management, Brainware University, Kolkata
Units (1-3, 5)
S. Kalavathy, Dean, Department of Science and Humanities, K. Ramakrishnan College of Technology, Samayapuram
Unit (4)

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• Website: www.vikaspublishing.com • Email: helpline@vikaspublishing.com

SYLLABI-BOOK MAPPING TABLE

Simulation and Modelling

Syllabi	Mapping in Book
UNIT-I: **System,** Introduction, System Study, System Examples.	**Unit-1:** System Study **(Pages 3-24)**
UNIT-II: **Modeling and Simulation-I,** System Modeling, system Simulation, Simulation and Modeling Process **Modeling and Simulation-II,** Introduction, Discrete System Models, Continuous System Models, Modeling and Simulation Platforms, Introduction, SIMSCRIPT, GPSS, CSMP III	**Unit-2:** Modeling and Simulation **(Pages 25-92)**
UNIT-III: **Model Verification and Validation,** Validation and Verification, Estimation Methods, Simulation Run Statistics, Replication of Runs, Regenerative Techniques	**Unit-3:** Model Verification and Validation **(Pages 93-107)**
UNIT-IV: **Monte Carlo Methods**, Introduction, Random Number Generation, Test for Randomness, An Application	**Unit-4:** Monte Carlo Methods **(Pages 109-160)**
UNIT-V: **Application of Simulation and Modeling**, Introduction, Application in Management, Application in optimization, Application in Artificial Intelligence, Application in Sociology, Application in Economics, Application in Life Sciences, Application in Database Designing, Application in Computer Designing	**Unit-5:** Application of Simulation and Modeling **(Pages 161-209)**

CONTENTS

INTRODUCTION

NOTES

Modelling and Simulation (M&S) is the use of models (e.g., physical, mathematical, or logical representation of a system, entity, phenomenon, or process) as a basis for simulations to develop data utilized for managerial or technical decision making.

A computer is used to generate a mathematical model that contains important parameters of the physical model in the computer application of modelling and simulation. The mathematical model is used to simulate the physical model, and circumstances are applied to set up the desired experiment. The simulation begins (i.e., the computer calculates the outcomes of those conditions on the mathematical model) and produces results in a machine- or human-readable format, depending on the implementation.

Simulation is the imitation of the operation of a real world process or system over time. The act of simulating something first requires that a model be developed; this model represents the key characteristics or behaviors/functions of the selected physical or abstract system or process. The model represents the system itself, whereas the simulation represents the operation of the system over time. Key issues in simulation include acquisition of valid source information about the relevant selection of key characteristics and behaviours, the use of simplifying approximations and assumptions within the simulation, and fidelity and validity of the simulation outcomes.

Monte Carlo methods or Monte Carlo experiments are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results; typically one runs simulations many times over in order to obtain the distribution of an unknown probabilistic entity. Monte Carlo methods are mainly used in three distinct problems classes: optimization, numerical integration and generation of draws from a probability distribution.

The application of M&S in engineering is well known. Engineers of all application fields use simulation technology, and it has been included in the body of knowledge of engineering management. M&S assists in cost reduction, improving product and system quality, and documenting and archiving lessons learned. Engineers, operators, and analysts must pay special attention to the simulation’s development because the outcomes are only as good as the underlying model(s). The user must grasp the assumptions, conceptualizations, and limits of the simulation’s implementation in order to verify that the simulation’s conclusions are appropriate in the real world. Models can also be updated and improved utilizing data from real-world experiments. M&S is a distinct discipline. Because of its diverse application fields, M&S is frequently mistaken for a pure application. This is not the case, and engineering management must recognize this when applying M&S.

NOTES

This book, *Simulation and Modelling*, follows the SIM format wherein each Unit begins with an Introduction to the topic followed by an outline of the 'Objectives'. The detailed content is then presented in a simple and an organized manner, interspersed with 'Check Your Progress' questions to test the understanding of the students. A 'Summary' along with a list of 'Key Terms' and a set of 'Self-Assessment Questions and Exercises' is also provided at the end of each unit for effective recapitulation.

UNIT 1 SYSTEM STUDY

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Structure

1.0 INTRODUCTION

Systems simulation is a set of techniques that uses computers to imitate the operations of various real-world tasks or processes through simulation. Computers are used to generate numeric models for the purpose of describing or displaying complex interaction among multiple variables within a system. The complexity of the system arises from the stochastic (probabilistic) nature of the events, rules for the interaction of the elements and the difficulty in perceiving the behavior of the systems as a whole with the passing of time. Modelling and simulation, precisely known as M&S in an abbreviated form, consists of either a physical or a logical illustration of a assumed or prearranged system and is used to engender data and assist in determining or concluding decision/decisions or yield predictions about the assumed or prearranged system. Modelling and simulation is extensively accepted and very much associated in the area of manufacturing, production, operations, social & physical sciences, development of various products, several areas of engineering and other areas.

In this unit, you will learn about the system, system study and system examples.

1.1 OBJECTIVES

After going through this unit you will be able to:

- Understand the basic of system
- Explain the significance of system study
- Illustrate the various examples of system

1.2 INTRODUCTION TO SYSTEM

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In the perspective of Modelling & Simulation, there are two most important terminologies. They are as follows:

- **System**: A system is an organized group of related objects that exists in an environment and functions in space and time.
- **Model**: A model is a abridged depiction of a system at some precise point in time or space envisioned to establish a transparent understanding of a tangible system.
- **Simulation**: A simulation is an abridgement of a specific model which is performed in such a way that it functions and behaves on time or space to wrap it, thus empowering one to distinguish the connections, communications, relations or interactions that would not else be ostensible because of their departure in time or in space.

Modelling and Simulation is a speciality which is referred for emerging a level of understanding the insight of numerous connections, communications, relations or interactions of a specific system. It can be a part of the system or can also be the system as a whole. The stage of understanding which may be generated by this discipline is almost impossible to achieve through any other means.

A system can be defined as an entity which establishes its existence via the various interaction of its parts. A model is a shortened or abridged depiction of the real system which is proposed to indorse understanding. A success of the model certainly rests on the degree to which it encourages understanding. Since all the models are considered to be the simplified version the real model, there is continually a trade-off as to what extent of particularise is considered in the model. If very less volume of details is considered in the model which is taken into consideration, then there is certainly a risk which will enable failure to appropriate consideration of interactions and the considered model will complete fail in generating appropriate understanding. On the other side, if too much aspects are comprised in the model, there is a possibility that the model may become exceedingly complex and actually preclude the development of understanding.

A simulation, in general, denotes to a computerized edition of the model which is executed in curse of time to learn the insinuations of the specified interactions. Simulations are normally and certainly iterative in nature during its deployment and development. Conventionally, an individual creates a model, simulates the developed model, observers from the corresponding simulation, amends the model as appropriate and as per specification or SLA and further carries out the iterations until a satisfactory and acceptable level of interpretation is generated.

Modelling and Simulation is certainly a kind of regulation or discipline. It is also a form of an art. One can theoretically learn how to drive a car by reading a book. But until and unless one physically learns the art of driving a car, the knowledge is not at all complete. In reality, modelling and simulation follows the same path. One can learn the concept of modelling and simulation by reading a book but until and unless it is practiced, hands-on, the concept can't be achieved.

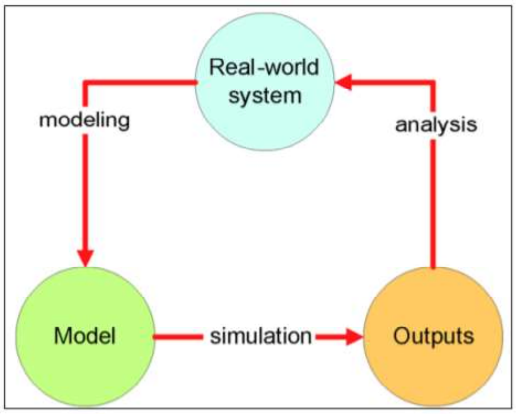

Fig. 1.1 *Flow of Modelling*

Figure 1.1 depicts the flow of modelling and simulation into the production environment.

Following are some of the examples of applications in the area modelling and simulation:

- Generating various complex models of weather forecasting systems, simulating behaviour with reference to the obtainable or existing data to produce prognostic report for weather forecasts. For example, a rainstorm or cyclone forecast model is premeditated or illustrated to forecast a specified storm's trail and might, as well as correlated proceedings such as surges generated along with the storm.
- Simulating the consequence of unembellished weather proceedings like tornados and hurricane gushes on infrastructure to lead the design of additional buoyant systems.
- Establishing a program to prototype a societal condition and witnessing the performance of entities in the simulation when the sequencer gets executed. Social simulations can be used to generate extrapolative data about how occurrence take place in actual-world environments. This is how the societal related standards are developed. Social simulation is the analysis of computer-based practice and technologies to simulate human societal conducts.
- Simulating the process of how a corporeal change to a system will affect its associated performance. For example, NASA has discovered drone aerodynamics with the help of the logical models by simulating air pressures and currents around the rotors. This is the information which can be utilized in the development of designing the process to reduce the turbulence, which would certainly make the vehicles more silent during operation.
- Via the simulation and modelling, network simulation can be generated for planning the testing of the disaster recovery.

A system is an abridged depiction of certainty. The word 'System' is frequently used with unrestricting meaning. Let us refer to a system as

- An array of identified or selected items or entities.

- It possesses a distinct and quantified boundary or boundaries.
- Consisting of prearranged and pre-programmed time features.

History of Simulation

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The history of simulation is as follows:

- 1940: A technique or process termed as 'Monte Carlo' was established by various scientists and physicists (John von Neumann, Stanislaw Ulan, Edward Teller, Herman Kahn). These scientists and researchers were, then, working on a project named 'Manhattan". This project was initiated to find out the nature of neutron scattering.
- 1960: SIMSCRIPT was developed by Harry Markowitz at the RAND Corporation. At that time it was considered to be the first unique simulation languages.
- 1970: During this time frame scientists and researchers initiated simulations, based on mathematical and statistical concepts.
- 1980: During this time frame, computer-based simulation software, Graphical User Interfaces (GUI) and Object-Oriented Programming (OOPS) were developed. All these were the triggers to generate a more developed simulation program.
- 1990: During this time frame, web-based simulation, decorative animated graphics, simulation-based optimization and Markov-chain Monte Carlo methods were developed.
- 2000: Machine learning and artificial intelligence were introduced to incorporate them within the simulation programs.

1.3 SYSTEM STUDY

In very simple terms, a system is a systematized assortment of subsystems that are exceedingly cohesive to achieve a complete task. Normally a system can have either one or various inputs, which goes into the system, the inputs are processed to yield a definite output, which in turn, achieve the complete anticipated goal for the specific system.

Normally, a system consists of various sub/smaller systems. For example, an organization consists of various functionalities like Sales, Marketing, HR, Systems, Production, accounts, etc. These departments are all sub systems or smaller systems which, once combined makes up the main system. In case one portion of the part of the sub-system is modified, there is a possibility that the overall system will have an impact. If case, the main system is changed, there will be a ripple effect on the sub system as well.

A System may be either simple or complex. We can consider this to be a range. There can be various types of systems. For example, our physiological systems, mechanical systems, banking system, transportation system, etc.

Now, let us consider a Complex system. Social systems can be considered to be a complex system. Social system consists of various subsystems like other systems. These subsystems are organized in hierarchical manner and are coupled together to achieve the general target of the overall system. Each subsystem within the main system has its own borders and restrictions. It consists of innumerable inputs, processes and outputs to achieve an overall goal for the said subsystem. Complex systems normally intermingle with its associated environments and are thus considered to be an open system.

A high-functioning system recurrently exchanges response within its various slices to guarantee that they persist thoroughly associated and attentive in attaining the goal of the system. In case, any portion of the established events in the system gets enfeebled or depleted, the associated system makes obligatory alterations or adjustments in a more effective manner to achieve the desired goals. Therefore, a system is always systematic in terms of its functionality.

A mass of sand is not considered to be a system. In case a grain is removed from the heap there still exists a heap of sand.

But, a functioning of a vehicle is considered to be a system. In case the wheel or the steering is removed the car will no longer function.

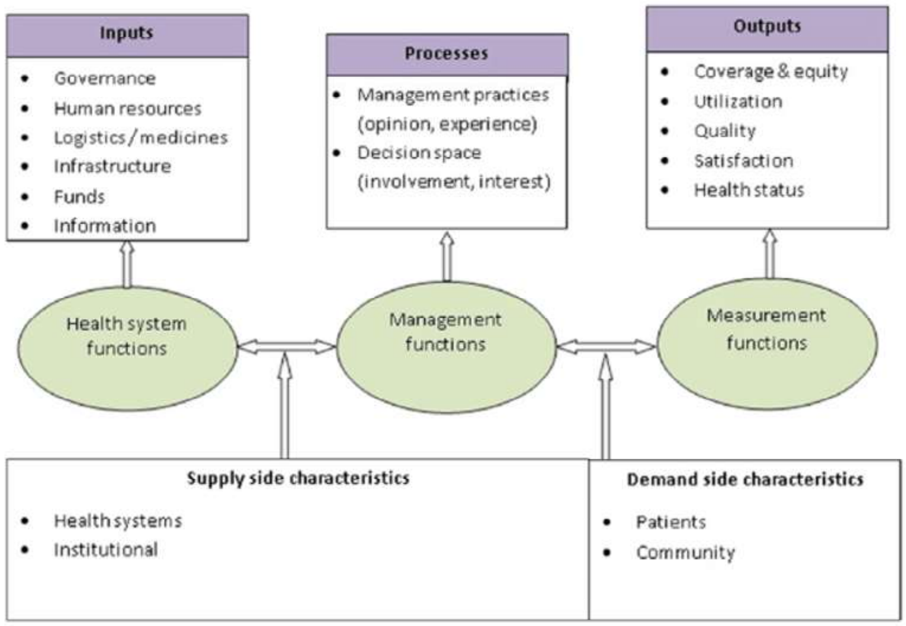

Fig. 1.2 *Relationship Among Inputs, Processes and Outputs through Supply and Demand-Side Factors*

Inputs

Inputs are considered to be the raw materials which are feed in to the system to achieve an output. Input is considered to be a mandatory factor in a system. If there are no inputs, the system will not be able to produce output. General types of inputs consist of human, currency, apparatus, amenities, materials, ideas by human being, etc. For example, inputs to a facility that offers training to clients ideally should include experienced and knowledgeable faculties, participants, training courseware, training rooms, labs, training amenities, etc. Inputs are considered to be the foremost forces that impacts the company and its products and services.

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For example, numerous laws and regulations impact how the products or services are offered. Inputs are frequently recognized with the price to attain and practice them. To understand it easily, a budget is a catalogue of the system's inputs and the associated expenditures to attain and utilize the inputs, along with any excises anticipated to be received or elevated (i.e., revenues) from the generated output from the system.

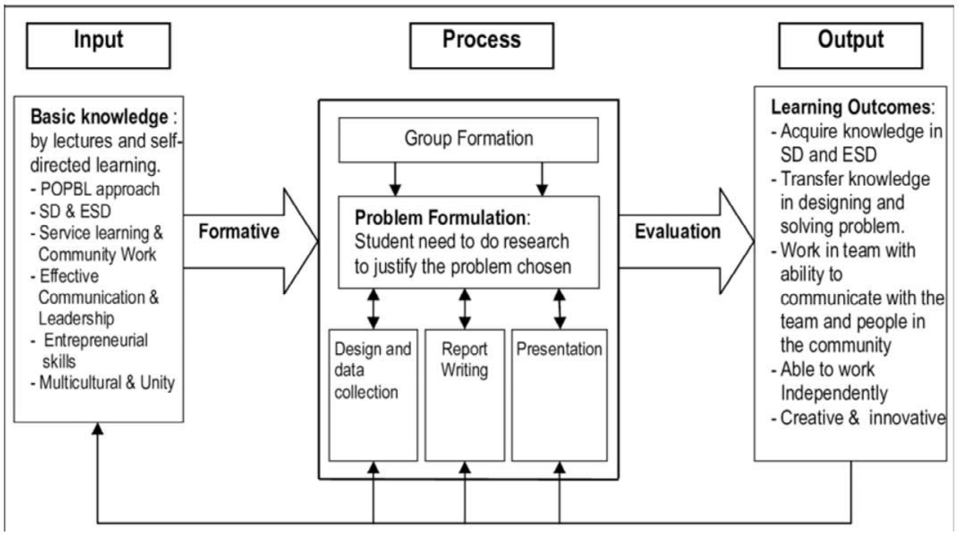

Fig. 1.3 *Input, Process and Output of Life Sciences in Education*

Processes (Procedures to yield the outcomes)

Process is either an activity or an array of activities which is performed in a sequential manner to produce a desired output. Unlike input, a process is also very important aspect of a system in terms of producing the desired output. Processes are highly influenced by the type of inputs it receives. Hence the inputs have to be accurate. For example, during the generation of balance sheet, the processes are defined based on the inputs. If the inputs are correct and the process if not correct then the desirable output will be completely incorrect. For example, the foremost processes used by a service which caters training to clients ideally consists of enrolment of students, admission testing, training, examination and providing certification. Processes can certainly vary from the simple state of placing a piece of paper on the table to complex manufacturing of steel bars. With an organization, the senior management or the leadership team normally defines and designs the process so that the desired outcome is flawless and as per compliance. Processes can also be termed as 'Activity/Activities,' 'Method/Methods', 'Techniques' or 'Throughputs.'

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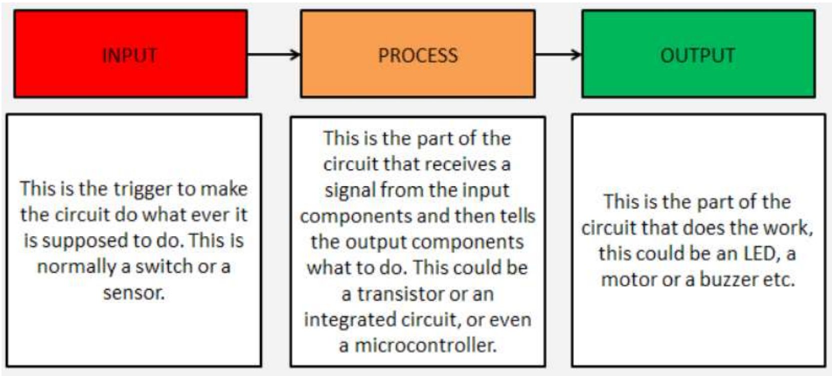

Fig. 1.4 *The Process Section of a Electronic Project*

Outputs (Tangible Results)

Outputs are considered to be the tangible results which are produced via the processes of an organization in terms of products or services. Outputs are normally as the end product post processing. For example, it can be the manufactured item i.e., a car manufacturer will produce car. Output can also be a service i.e., providing IT support to an organization.

Outputs and success of an organization are completely different.

The success of a product or services is completely dependent on the customer's satisfaction and not on the various inputs and processes used to create the product. The success of a product or service completely depends on how well the consumer or customer are benefited.

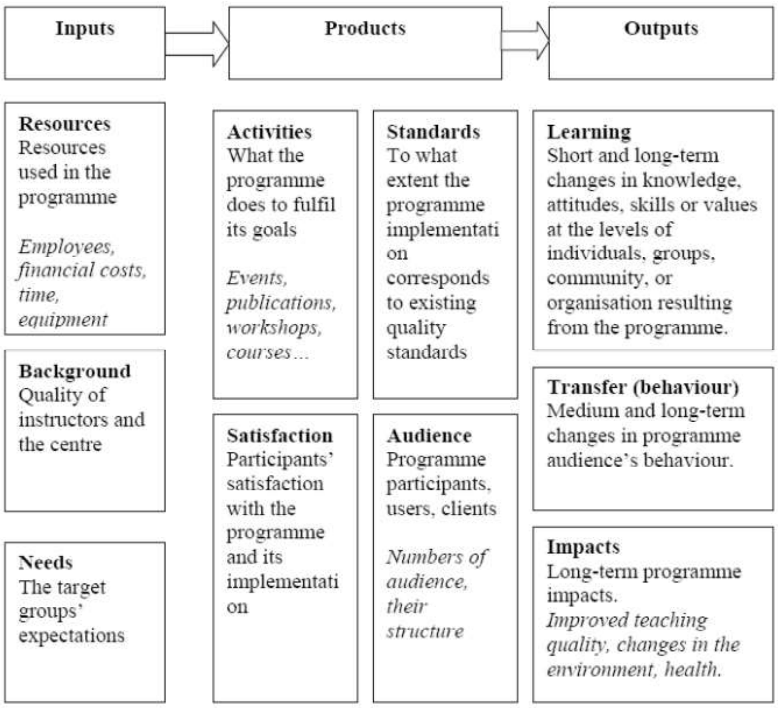

Fig. 1.5 *Input-Process-Output Model*

Goals and Outcomes (Desired Results)

Goals are the final resultant which the system opts to achieve. Generally, all systems in the universe are goal-oriented.

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For example, an automobile manufacturing organization is responsible to manufacture vehicles of specific type and purpose.

To obtain desired results, an organization and its branches/subsidiaries should have well defined goals. To achieve the desired results, the senior management/ leadership team should draft the goals clearly and should communicate the same employees as appropriate so that each and all employees are aware about their company's target. By doing this, it becomes easier for all employees to follow the process to obtain the goal successfully.

The general goals of an organization are typically described in terms of its mission, vision and value statement.

A mission statement reveals the organization's purpose for being, and how it intends to serve its major stakeholders. Stakeholders are the consumers/ customers, employees, and investors of the organization.

Values are the confidences of an organization. The Starbucks mission statement pronounces six guiding principles that, at the same time communicates the organization's values:

- Provide a great work environment and treat each other with respect and dignity.
- Embrace diversity as an essential component in the way we do business.
- Apply the highest standards of excellence to the purchasing, roasting and fresh delivery of our coffee.
- Develop enthusiastically satisfied customers all of the time.
- Contribute positively to our communities and our environment.
- Recognize that profitability is essential to our future success (Starbucks, 2008).

Similarly, Toyota declares its global corporate principles to be:

- Honour the language and spirit of the law of every nation and undertake open and fair corporate activities to be a good corporate citizen of the world.
- Respect the culture and customs of every nation and contribute to economic and social development through corporate activities in the communities.
- Dedicate ourselves to providing clean and safe products and to enhancing the quality of life everywhere through all our activities.
- Create and develop advanced technologies and provide outstanding products and services that fulfil the needs of customers worldwide.
- Foster a corporate culture that enhances individual creativity and teamwork value, while honouring mutual trust and respect between labour and management.

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- Pursue growth in harmony with the global community through innovative management.
- Work with business partners in research and creation to achieve stable, long-term growth and mutual benefits, while keeping ourselves open to new partnerships (Toyota, 2008).

A vision statement, in difference, is a future-oriented assertion or announcement of an organization's resolution and objectives.

Normally, vision statements are comparatively concise and succinct. For example, Starbuck's vision statement, is as follows:

'Establish Starbucks as the premier purveyor of the finest coffee in the world while maintaining our uncompromising principles as we grow (Starbucks, 2008).'

Ogilvy & Mather asserts their vision as 'an agency defined by its devotion to brands (Ogilvy, 2008).'

Mission and vision statements play three critical roles:

I. To convey the resolution of the organization to the stakeholders.

II. Convey the developed strategies.

III. Create or identify the quantifiable goals and objectives of the organization which is to be used or referred to during measuring the achievement and accomplishment of the organization's approach or strategy.

These inter-reliant drifting roles and the corresponding relationship among them, are provided in the Figure 1.6.

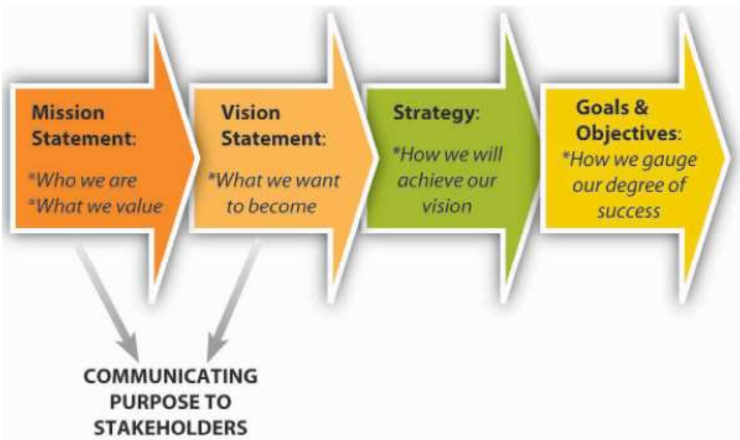

Fig. 1.6 *Key Roles of Mission and Vision*

Some examples of mission, vision and value statement of renowned organization are as follows:

Max Health is a supplement organization which produces healthcare products and supplements. The organization mainly focuses on healthy bodies, minds and items that supplement getting mentally or physically strong.

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- Mission statement: Dedicated to personal physical and mental fitness for people of all ages.
- Vision: Everybody matters, and every mind needs the same level of care.
- Values: Fitness, mental health, personal care, integrity and progress

An organization can certainly consider its goals in a various dimension, for example, the organization can consider its goals in relation to its activities, activities of the consumers or impacts on the consumers. These impacts are normally named as the outcomes.

Feedback

Feedback is the process of continuous exchange of opinions about a product. Feedbacks can also be about numerous departments of an organization as well. Feedback can be coming from various sources for example, from the external stakeholders (i.e., consumers, community front-runners, investors, etc), the board members, the leadership team, the CEO of an organization and other employees. Feedback can also be obtained from evaluating various products, various services or individuals. Collection of feedbacks is a ongoing process and very much essential as well as critical for an organization to succeed in the future.

Assessment and Evaluation

Assessment is the process of performing certain measurement from the collected feedbacks from clients or consumers. Evaluation is the process which consists of gathering various relevant information (it can be feedbacks as well), in a methodical style and concluding to formulate the desired optimal decisions.

A professional organization always keeps on collecting and assessing feedback (360-degree feedbacks) to assess the efficiency of the organization. Feedbacks are collected internally and externally as well. Normally, the assessment and evaluation are engrossed on numerous outputs. They also measure feedbacks from the system as well. Evaluation is either focused on the complete organization or can also be on any specific branch or cost centre. For example, feedbacks can be collected from customer directly to evaluate their opinion about the product. This might also contain suggestions about any kind of specific improvement. Feedbacks can also be collected internally to find out in case the internal system of the organization is working perfectly or not in terms of quality or compliance.

Learning

To achieve desired result, it is essential to attain enhanced knowledge. Knowledge will certainly help enhancing the skills and attitudes to accomplish an activity successfully. Learning can take place when and individual is within a group or within groups or within the organization itself. Acquiring knowledge is very much critical as it takes into consideration that the company is continually enlightening its interpretation of itself and its associated environment.

System Approach

According to Jackson, the systems method to engineered systems is a problem-solving standard. It is a complete problem identification and resolving method founded upon the philosophies, ideas and various tools of system thinking and systems science, along with the notions integral in engineering problem-solving. It includes a complete systems assessment that consists of the bigger framework of the system, including engineering and operational atmospheres, stakeholders and the complete life cycle.

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Check Your Progress

1. Define the term model.
2. What is system?
3. Define inputs in a system.
4. Define the term output in system.
5. How will you define the assessment and evaluation?

1.4 SYSTEM EXAMPLES

Systems and processes are the structural and functional unit of an organization. Every aspect of a business—in a specific department, in the storeroom or in the factory—is part of a system that can be controlled and optimized by applying correct procedures.

'Business system' consists of two words i.e., Business & System.

A system is a combination of policies, procedures, people and equipment to coordinate the activities of an organization. Business system establishes the process and procedures of an organization.

'Business system' decides and concludes how business data must be handled and processed systematically. It also establishes controls over the procedures of the processed data and its corresponding result. E.g. a system may automatically crate purchase order for spare parts, monitor profits or perform postings of transactions. The overall nature of the business system should reflect the efficiency of its designers via the established technology.

A business system is fabricated to unite all of an organization's various parts and interrelated steps to work collectively for the achievement of the defined business goal. These systems and processes which can be termed as the foundation elements work in a hub and spoke topology to achieve the business goal in a successful manner.

The main purpose of the business systems and processes are to:

- Meeting customer's expectations
- Customer satisfaction
- Consistent Results
- To identify correct and suitable technique of handling business activities.
- Employee engagement
- Cost control
- Increase Profits

- Eliminate redundant process
- Flow of accurate data among all departments
- Speed up execution of processes accurately

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Type of Business Systems

Business Systems are inclusion of a compilation of adjacent and contiguous business processes associating workflows collectively to offer a worth proposition that results the broader objective of the Business System itself. “System” includes a collection of adjacent and contiguous business processes connecting workflow together to provide a worth proposition that delivers the broader objective of the business system itself. Business Systems are commonly recognized by its department names, such as Sales, HR, Procurement or Finance. The size and depth of each business system may differ by organization, but the basic workflow, connectivity and association remains the same.

Examples of Business System

- **Enterprise Management System**: Defines organization strategy, establishes the organization’s business scorecard, executes management activities and scrutinize organization performance via planned management review.
- **Financial Management System**: It manages and keeps an eye on the flow of funds through the organization, including financial transactions and accounting. Some of the activities of this department are as follows:
  - Financial decisions making and financial controls
  - Financial Planning
  - Capital Management
  - Allocation and Utilization of financial resources
  - Cash Flow Management
  - Disposal of Surplus
  - Financial Reporting
  - Risk Management
  - Generation Financial reports
  - Handling financial audits
- **Facilities Management System**: Maintains organization facilities to provide an appropriate working environment. Some of the activities of this department are as follows:
  - Property management & strategy
  - Office Space management
  - Communications infrastructure (Speaker, CCTV, TV, etc)
  - Office Building maintenance
  - Testing and inspections of the facilities
  - Office Building administration

- Contract management (with vendors)
- Environment, Health, Safety (EHS)
- Office building Security
- Business continuity planning
- Management of office building renovations and refurbishments

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- **Maintenance Management System**: This function deploys calibrates and maintains equipment (e.g., AC plant maintenance, Factory machinery maintenance, etc.) utilized to cater the organization's value proposition. Some of the activities of this department are as follows:
  - Scheduling and overseeing all maintenance-related work by managing a team of maintenance technicians and supervisors
  - Ensuring that all maintenance operations are done in accordance with company policy and defined standard guidelines
  - Ensuring the facility fulfils all industry regulations
  - Forecasting, ordering, and price negotiation for spare parts inventory
  - Developing and implementing a (proactive) maintenance program
  - Hiring of maintenance employees
  - Coordinating the completion of repairs
  - Hiring subcontractors for maintenance work (specialized)
  - Looking for new ways/tools/assets to improve productivity and cut costs
  - Drafting maintenance reports
  - Drafting policies and procedures for maintenance related activities
- **HR/Employee Management System**: This system hires, monitors, develops, transfers and terminates employees. Some of the activities of this department are as follows:
  - Human resource planning
  - Recruitment and selection
  - Defining KRA (Key responsible area/KPI(Key performance indicator)
  - Performance management
  - Learning & development
  - Payroll management
  - Career planning
  - Function evaluation
  - Rewards & Recognition
  - Employee participation and communication
  - Health & safety
  - Personal wellbeing
  - Administrative activities

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- **Information Management System**: This division maintains the hardware, software and data related activities as per business requirements. Some of the activities of this department are as follows:
  - Manage information technology and computer systems
  - Plan, organize, control and evaluate IT operations
  - Management of IT staff - recruiting, training, monitoring and appraising their performance
  - Develop and implement policies and procedures
  - Ensuring of data security, network access and backup systems
  - Act in alignment with business needs and system activities to contribute to organizational policy
  - Identify gaps and implement permanent solutions
  - Auditing systems and access their outcomes
  - Preserving of assets and control structures
  - Handling of annual budget and ensuring cost effectiveness
- **Customer Development System**: This division is mainly responsible for sales and marketing. Some of the activities of this department are as follows:
  - To promote the organization's existing products and introducing new products to the market.
  - To ensure that the sales team meets their targets.
  - R&D of marketing opportunities and plans, understanding the consumer requirements, identifying the market trends, and suggesting improvements to achieve the organization's marketing aim.
  - Analyze sales data and act accordingly.
  - To implement new sales plans and advertisements.
  - Recruiting, training, scheduling, and managing of sales team.
  - To maintain professional relationship with important clients by making regular visits, understanding their requirements and anticipating fresh marketing requirements/opportunities.
  - Staying up-to-date in the industry by attending training, conferences, and workshops and certifications.
- **Product Development System**: This division is responsible for inventing, planning, developing, testing and delivering of new products and services in the market as well as discarding the previous active products and services. In gist this division is responsible for controlling the complete life cycle of a particular product. Some of the activities of this department are as follows:
  - Create and improve existing products of the organization
  - Determine existing product specifications and propose for improvement
  - Development of prototypes

- Consult with engineers/supervisors
- Review market research for products of other companies.

- **Quality Management System**: Quality management ensures that an organization, product or service is consistent. This activity has four main components: Quality planning, Quality assurance, Quality control and Quality improvement. Some of the activities of this department are as follows:
  - Reading of blueprints and specifications
  - Monitoring of operations to ensure that they meet the conformed standards
  - Recommending of improvements to the existing process
  - Testing of products being produced
  - Accepting/rejecting of finished products
  - Removing/Trashing of all products/materials that fail to meet conformed specifications
  - Preparing management dashboards for quality conformance.
  - Approval of all finished conformed materials
  - Document and update inspection results
  - Maintain safety environment by following conformed standards and complying with legal requirements (statutory & regulatory)
  - Pertain training to junior employees
  - Conducting organization-wise quality awareness
  - Participate in quality control audit

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Business Intelligence (BI)

Business Intelligence (BI) plays a major role in this regards. Using business intelligence, business facts are analyzed and the concluding resultants are channelized through optimized processes to achieve business goal. In more lucid term, Business Intelligence (BI) permits businesses to learn about the existing process or drift influencing performance, reason things are occurring, and what is to transpire in the future.

Classification of system

Kenneth Boulding (1956) is considered to be the fathers of general system theory. He developed a systems classification theory, which, is referred to by many scientists and researchers for various system related subsequent research and work. Boulding, categorized system into nine types. They are:

- Structures (Bridges)
- Clock works (Solar system)
- Controls (Thermostat)
- Open (Biological cells)
- Lower organisms (Plants)

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- Animals (Birds)
- Man (Humans)
- Social (Families)
- Transcendental (God)

In the year 1999, Peter Checkland proposes the categorization of a system systems into five classes. They are:

- Natural systems,
- Designed physical systems,
- Designed abstract systems,
- Human activity systems
- Transcendental systems

According to Checkland, the concatenation of the above stated five systems are referred to as "systems map of the universe".

The following figure depicts a overall interpretation of the context for any probable application of an engineered system life cycle.

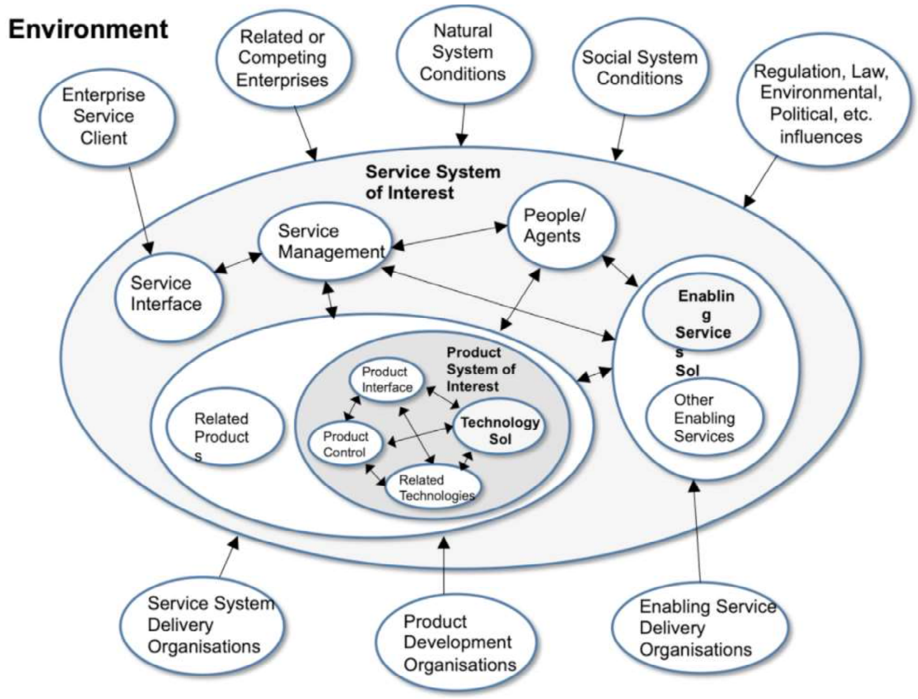


Fig. 1.7 *Service System of Interest*

From the Figure 1.7, we can conclude the following points:

- A technology attentive product system SoI entrenched within one or supplementary cohesive products.
- A combined multi-technology product system SoI which is operated straight away to assist in providing a specific service.
- An empowering service system SoI which is used to support multiple other service systems.

- A service system SoI formed and continued to straight away deliver certain competence.

There can be various kinds of systems and they are as follows:

- Linear and Non-linear Systems
- Time Variant and Time Invariant Systems
- Linear Time variant and linear Time invariant systems
- Static and Dynamic Systems
- Causal and Non-causal Systems
- Invertible and Non-Invertible Systems
- Stable and Unstable Systems

Linear and Non-linear Systems

A system is considered to be linear when it fulfils the superposition and homogenate principles. Let us consider two systems with inputs as c1(t), c2(t), and outputs as d1(t), d2(t) respectively. So, as per the superposition and homogenate principles,

$T\,[a1\ c1(t) + a2\ x2(t)] = a1\ T[c1(t)] + a2\ T[c2(t)]$

4", $T\,[a1\ c1(t) + a2\ c2(t)] = a1\ d1(t) + a2\ d2(t)$

From the above expression, is understandable that the response of overall system is equal to response of individual system.

Example:

$(t) = x2(t)$

Solution:

$d1\ (t) = T[c1(t)] = c12(t)$

$d2\ (t) = T[c2(t)] = c22(t)$

$T\,[a1\ c1(t) + a2\ c2(t)] = [\ a1\ c1(t) + a2\ c2(t)]2$

Which is not equal to a1 d1(t) + a2 d2(t).

As this is not equal, the system is said to be non-linear.

Time Variant and Time Invariant Systems

A system is considered to be time variant if its input and output characteristics differ with respect to time. Else, the system is said as time invariant.

The condition for time invariant system is:

$y\,(n\,,\,t) = y(n-t)$

The condition for time variant system is as follows:

y (n , t) ‘Š” y(n-t)

Where $y\,(n\,,\,t) = T[x(n-t)]$ = input change

$y\,(n-t)$ = output change

Example:

$y(n) = x(-n)$

$$y(n, t) = T[x(n-t)] = x(-n-t)$$

$$y(n-t) = x(-(n-t)) = x(-n + t)$$

$$\therefore y(n, t) \neq y(n-t)$$

Hence, the system is considered as time variant.

Linear Time Variant (LTV) and Linear Time Invariant (LTI) Systems

If a system is equally linear and time variant, then it is known as Linear Time Variant (LTV) system.

If a system is equally linear and time invariant then that system is known as Linear Time Invariant (LTI) system.

Static and Dynamic Systems

In nature, Static system is memory-less whereas dynamic system is a memory system.

For example, $y(t) = 2\,x(t)$

For current value t=0, the system output is $y(0) = 2x(0)$.

Here, the output is only reliant on the current input. Hence the system is memory less or static.

For example, $y(t) = 2\,x(t) + 3\,x(t-3)$

For present value t=0, the system output is $y(0) = 2x(0) + 3x(-3)$.

Here $x(-3)$ is previous value for the current input for which the system requires memory to get this output. Hence, the system is a dynamic system.

Causal and Non-Causal Systems

A system is considered to be causal in case its output reliant upon the current and previous inputs, and, at the same time, is not reliant or depend upon future input.

For non-causal system, the output depends upon future inputs as well.

For example, $y(n) = 2\,x(t) + 3\,x(t-3)$

For present value t=1, the system output is $y(1) = 2x(1) + 3x(-2)$.

In this case, the system output solely depends upon current and previous inputs. So, the system is considered to be causal.

For example, $y(n) = 2\,x(t) + 3\,x(t-3) + 6x(t + 3)$

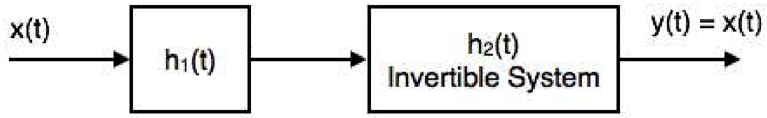
For the current value t =1, the system output is $y(1) = 2x(1) + 3x(-2) + 6x(4)$

Here, the system output is dependent on future input.

In such case the system is non-causal system.

Invertible and Non-Invertible systems

A system is considered to invertible in case the input of the system appears at the output.



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Y(S) = X(S) H1(S) H2(S)

= X(S) H1(S) • 1(H1(S))1(H1(S))   Since H2(S) = 1/(H1(S))

4", Y(S) = X(S)

'! y(t) = x(t)

In such case, the system is considered to be invertible.

If y(t) 'Š" x(t), then the system is considered to be non-invertible.

Stable and Unstable Systems

The system is considered to be stable only when the output is bounded for bounded input.

For a bounded input, in case the output is unbounded in the system then it is considered to be unstable system.

For a bounded signal, the amplitude is always finite.

For example, y (t) = x2(t)

Let the input is u(t) (unit step bounded input) then the output y(t) = u2(t) = u(t) = bounded output.

In such case, the system is considered to be stable.

For example, y (t) = +"x(t)dt

Let the input is u (t) (unit step bounded input) then the output y(t) = +"u(t)dt = ramp signal (unbounded because amplitude of ramp is not finite it goes to infinite when t '! infinite).

In such case, the system is considered to be unstable.

Check Your Progress

6. Write the purpose of the business systems and processes.
7. Write the examples of business system.
8. What is the role of Business Intelligence (BI)?
9. When will the system is considered astime invariant?

1.5 ANSWERS TO 'CHECK YOUR PROGRESS'

1. A model is a abridged depiction of a system at some precise point in time or space envisioned to establish a transparent understanding of a tangible system.
2. A system is a systematized assortment of subsystems that are exceedingly cohesive to achieve a complete task. Normally a system can have either

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one or various inputs, which goes into the system, the inputs are processed to yield a definite output, which in turn, achieve the complete anticipated goal for the specific system.

3. Inputs are considered to be the raw materials which are feed in to the system to achieve an output. Input is considered to be a mandatory factor in a system. If there are no inputs, the system will not be able to produce output.
4. Outputs are considered to be the tangible results which are produced via the processes of an organization in terms of products or services. Outputs are normally as the end product post processing.
5. Assessment is the process of performing certain measurement from the collected feedbacks from clients or consumers. Evaluation is the process which consists of gathering various relevant information (it can be feedbacks as well), in a methodical style and concluding to formulate the desired optimal decisions.
6. The main purpose of the business systems and processes are to:
   - Meeting customer's expectations
   - Customer satisfaction
   - Consistent Results
   - To identify correct and suitable technique of handling business activities.
   - Employee engagement
   - Cost control
   - Increase Profits
   - Eliminate redundant process
   - Flow of accurate data among all departments
   - Speed up execution of processes accurately
7. Examples of business system are:
   - Enterprise Management System
   - Financial Management System
8. Business Intelligence (BI) plays a major role. Using business intelligence, business facts are analyzed and the concluding resultants are channelized through optimized processes to achieve business goal. In more lucid term, Business Intelligence (BI) permits businesses to learn about the existing process or drift influencing performance, reason things are occurring, and what is to transpire in the future.
9. A system is considered to be time variant if its input and output characteristics differ with respect to time. Else, the system is said as time invariant.

1.6 SUMMARY

NOTES

- Modelling and simulation, precisely known as M&S in an abbreviated form, consists of either a physical or a logical illustration of a assumed or prearranged system and is used to engender data and assist in determining or concluding decision/decisions or yield predictions about the assumed or prearranged system.
- A system is an organized group of objects that exists in an environment and functions in space and time.
- A model is a abridged depiction of a system at some precise point in time or space envisioned to establish a transparent understanding of a tangible system.
- A system is a systematized assortment of subsystems that are exceedingly cohesive to achieve a complete task.
- Inputs are considered to be the raw materials which are feed in to the system to achieve an output.
- Process is either an activity or a array of activities which is performed in a sequential manner to produce a desired output.
- Outputs are considered to be the tangible results which are produced via the processes of an organization in terms of products or services.
- Goals are the final resultant which the system opts to achieve. Generally, all systems in the universe are goal-oriented.
- Feedback is the process of continuous exchange of opinions about a product.
- A system is considered to be causal in case its output reliant upon the current and previous inputs, and, at the same time, is not reliant or depend upon future input.

1.7 KEY TERMS

- **System**: It is an organized group of related objects that words in and environment.
- **Model:** It is a abridged depiction of a system at some precise point in time or space envisioned to establish a transparent understanding of a tangible system.
- **Simulation**: It is an abridgement of a specific model which is performed in such a way that it functions and behaves on time or space to wrap it, thus empowering one to distinguish the connections, communications, relations or interactions that would not else be ostensible because of their departure in time or in space.
- **Causal:** A system is considered to be causal in case its output reliant upon the current and previous inputs, and, at the same time, is not reliant or depend upon future input.

1.8 SELF-ASSESSMENT QUESTIONS AND EXERCISES

NOTES

Short-Answer Questions

1. Define the term system.
2. Write the goals and outcomes of system.
3. What is the main purpose of the business systems and processes?

Long-Answer Questions

1. Discuss briefly about the system with the help of examples.
2. Explain briefly about the input-process-output model. Give appropriate examples.
3. Differentiate between the enterprise management system and financial management system with the help of examples.

1.9 FURTHER READING

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UNIT 2 MODELLING AND SIMULATION

NOTES

Structure

2.0 INTRODUCTION

System modelling is the procedure of building immaterial, intangible or abstract models of a specific system. Each model represents a diverse interpretation or standpoint of a particular system. System modelling normally represents a specific system via graphical representation. These representations are normally based on notations in the Unified Modelling Language (UML). In the realm of software engineering, the Unified Modelling Language (UML) is a general-purpose, developmental modelling language that is designed to provide a common way to depict a system's architecture.

Simulation is described as the imitation of the procedure of a real-life process or system for a given point of time. It is the procedure of investigating with a model of the process or system under analysis and it deals with the model of the procedure or system apart from than the original process or system itself. In the field of engineering, simulation is expressed as the procedure or method of investigating with a model of a specific system under research or survey by means of computer related applications or programming.

In this unit, you will learn about the system modelling, simulation and modelling process, discrete system models, continuous system models, modelling and simulation platforms, SIMSCRIPT, GPSS and CSMP III.

2.1 OBJECTIVES

After going through this unit you will be able to:

- Understand the basic of system modelling

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- Learn about the simulation and modelling process
- Elaborate on the discrete system models
- Define the continuous system
- Explain about the modelling and simulation platform
- Discuss about the SIMSCRIPT, GPSS, CSMP III

2.2 SYSTEM MODELLING

System modelling is the procedure of building immaterial, intangible or abstract models of a specific system. Each model represents a diverse interpretation or standpoint of a particular system. System modelling normally represents a specific system by via graphical representation. These representations are normally based on notations in the Unified Modelling Language (UML). At the same time, it is also possible to generate mathematical models of a specific system, as per detailed system requirement.

Models are normally used for the following purposes.

- At the time of the requirements engineering process to assists in deriving the necessities for a specific system.
- At the time of the design process or design phase to define the specific system to engineers who are responsible to implement the system.
- Post implementation phase to articulate to draft documents consisting of the system's structure and operational procedures.

Models can be developed for new systems as well as for the existing system based on the requirement:

1. Developing models of the existing system are required during the requirements analysis. This is used to analyse current specifications and based on the findings, improvement requirements can be evaluated and designed. Strengths and weaknesses of the existing system can also be evaluated from this kind of models. Post analysis, requirement for system modifications cab be evolved.
2. Models of the new system are referred to during requirements phase to assist in explaining the projected requirements to other system stakeholders. Engineers refer to these models for discussing and concluding the design proposals and to draft documentations of the system which is to be implemented. It is very much possible to implement a full or partial system via the model-driven approach.

The utmost importance of a system model is that it departs out the details. A model is a notion or abstraction of the system which is used for studying. It is not a substitute of the original system. In an ideal situation, a depiction of a system should preserve all the statistics about the object being represented but regrettably the real world (which is also known as the universe of discourse) is absolutely complicated so tidy to shorten. A notion intentionally streamlines and elects the maximum number of apparent features or characteristics of a specific system.

Various models can be developed to characterize the system from diverse perceptions or viewpoints.

For example:

- A model can be developed for an external perspective, where the context or environment of the system is exemplified or modelled.
- A model can be developed for the purpose of an interaction. In such case the model represents the interactions between a specific system and its associated environment or various constituents of the system.
- A model can be developed on the basis of structural perspective. This model consists of the composition of a system or the construction of the data that is managed by the system.
- A model can be developed on the behavioural perspective. This model captures the dynamic behaviour of the system along with the response details to the associated events.

These perspectives are very similar with Krutchen's 4 + 1 view of system architecture. In this architecture, Krutchen states that a system's architecture and design from various perspectives should be documented.

A flexible approach in terms of using graphical notation can be adapted while developing system models. It is not mandatory to stick rigidly to the particulars of the notations. The specification and thoroughness of a model depends on how you propose to use it. There are three ways in which graphical models are normally referred to. These are as follows:

- As a method of enabling conversation about a prevailing or projected system.
- As a method of documenting a present system.
- As a comprehensive system explanation that can be referred to while generating the implementation of a system.

In the first case, the intention of the model is to kindle the conversation between the engineers tangled in rising the system. The models might be inadequate and modelling notations can be used in an informal manner. This situation is known as 'Agile Modelling' and this is how the model is used or referred to.

In the second case, it is not mandatory to use a complete model as this is used for some parts of the system. But it is essential and mandatory that the model should be accurate. Correct notations should be used. There should be precise explanation of the system in this model.

In the third case, the model is used as portion of a model-based development procedure. In this case the system models must be comprehensive and precise. This model is used to generate the source code of the system and hence accuracy and completeness is a mandatory. Hence, confusing symbols should be avoided and the conventional method should be followed to create such model.

So, in GIST:

- System modelling is the procedure of generating a conceptual or abstract model of a specific system. Each model represents a diverse interpretation or standpoint of that specific system.

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- System modelling is a norm representation of a system which is accomplished by using certain of graphical representation. These graphical notations are based on symbolizations which is used in the Unified Modelling Language (UML).
- System modelling assists the implementers to comprehend the purpose of the system and models. They are also referred to while discussing with the clients.
- Models which are created on the basis of the existing system are referred to during the understanding of the requirements. This kind of model helps in understanding what is the current system does. It is also used to conclude the fortes and limitations of the current system. These are then used to conclude whether upgradation is required or not for the existing system.
- Models for new system are referred to during the understanding of the requirements which helps the implementers or engineers to elucidate the projected requirements to supplementary system stakeholders. Engineers refer to these models to converse design related proposals and to articulate the system for deployment.
- For a model-driven engineering process, it is very much feasible to spawn a comprehensive or fractional system employment from the primary system model.

The system perspectives are as follows:

- An external perspective: The context or environment of the system is modelled.
- An interaction perspective: The connections and communications between a system and its corresponding associated environment, or between the elements of an associated system is modelled.
- A structural perspective: The configuration of a system or the structure of the data that is managed by the system is modelled.
- A behavioural perspective: The vigorous conduct of the system and how it replies to actions is modelled.

We have spoken about the Unified Modelling Language (UML) diagram. The various types of UML diagrams are as follows:

- Activity diagrams: This kind of diagrams is used to demonstrate the various events or activities which are involved within a process.
- Use case diagrams: This kind of diagrams is used to demonstrate the various interactions among a system and its associated environment.
- Sequence diagrams: This kind of diagrams is used to demonstrate the interactions between the various performers and the system and between various system elements.
- Class diagrams: This kind of diagrams is used to demonstrate the object classes within the system and the relations between the corresponding classes.
- State diagrams: This kind of diagrams is used to demonstrate how the system ideally responds to internal and external consequences.

The benefits of using graphical models are as follows:

- As a method of simplifying conversation about a prevailing or projected system
- Unfinished and improper models are accepted as its main purpose is to assist in discussion.
- Used for drafting and documenting a prevailing system
  - o Models must be precise but it is not necessary to have a complete model.
- Should consists of a comprehensive system explanation as it will be used to spawn the implementation process of a system.
- In such case, the models have to be both accurate and comprehensive.

Defining system boundary is a very important thing in the area of modelling.

- System boundaries assists us in recognizing what is inside and what is outside the system.
- It depicts other systems which are used or dependent on the system which is already developed.
- The location of the system boundary has a intense consequence on the system necessities.
- Defining a system boundary is certainly a diplomatic decision
- There might be stresses to define system boundaries that rise / decline the effect or workload of various portions of an organization.

The various kinds of models are as follows:

- Context models
- Interaction models
- Structural models
- Behavioural models
- Model-driven engineering

Context Models

- Context models are used to demonstrate the operational framework of a specific system – it demonstrates what are laying outside of the system restrictions.
- Context models usually demonstrates the system's environment comprises of other systems but the categories of relations and associations between the systems of the environment and the system that is being stated are never demonstrated.
- In certain instances, the boundary in between the system and its environment is comparatively understandable.
- There may be a concern like the social and organisational distresses might influence the conclusion on where to spot the system boundaries.
- This model includes working with the various system stakeholders to conclude what functionalities should be encompassed within the system and what is offered by the system's own environment.

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- Architectural models demonstrate the system and its association with various other systems.
- Usually, generating an uncomplicated architectural model is the primary phase in this particular activity.

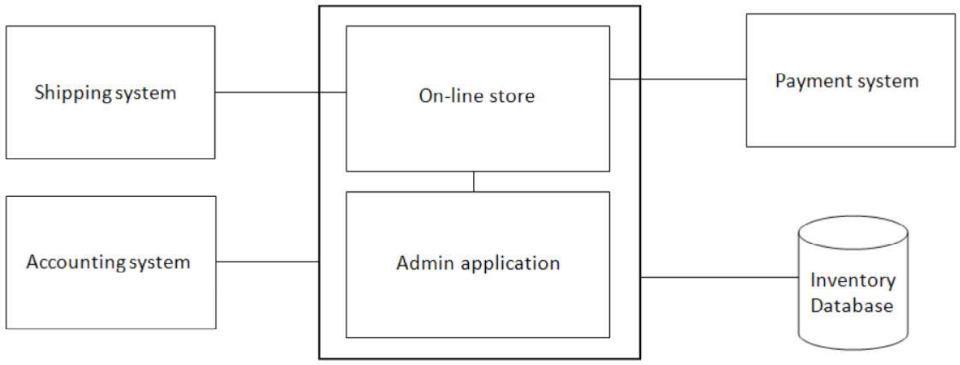

Fig. 2.1 *Example of a Context Model of the Online Store*

Figure 2.1 demonstrates an example of a context model. If we have a closer look at this model, it will be very easy for us to identify where the boundaries of this system exist. We can also identify the various functionalities belonging to the system and the functionalities which are external to the system. As a matter of fact, we have various kind of users with certain variant individualities and hence the system is fragmented into two different parts. They are:

- The online store itself (this is where the customers see). It consists of all the features and functionalities which is required by the consumers.
- An admin application which is used to perform the administrative activities.

The second activity not used very frequently. Moreover, there will be less users having administrative roles. That is the reason, this module has been kept separately.

Interaction Models

- To recognize and categorize the user requirements, user interaction model is very important.
- This mode is also used to identify the system-to-system interaction. Using this, we can identify the drawbacks in relation to the communication which might arise.
- Using this model, we can understand the various component used in the interaction. At the same time, we can also understand if a projected system structure is able to carry out the obligatory system performance and reliability.
- We can use the "Use case diagrams" and "Sequence diagrams" for interaction modellingas well.
- "Use case modelling" is used to prototype the various interactions between a system and external actors (i.e., users or any other system/systems).
- "Sequence diagrams" is used to prototype interactions between various system components. External agents are also included in such model.

- "Use case models" and "sequence diagrams" represents interaction at various levels of detail. Hence bot the diagrams can be used together.
- During interaction modelling, supplementary use cases can be generated. The context model should be referred to for identifying what other systems should the application interrelate or communicate with.
- "Actors" represents the part participated by normal physical users, external systems, hardware, software or any other Configurable Items (CI).

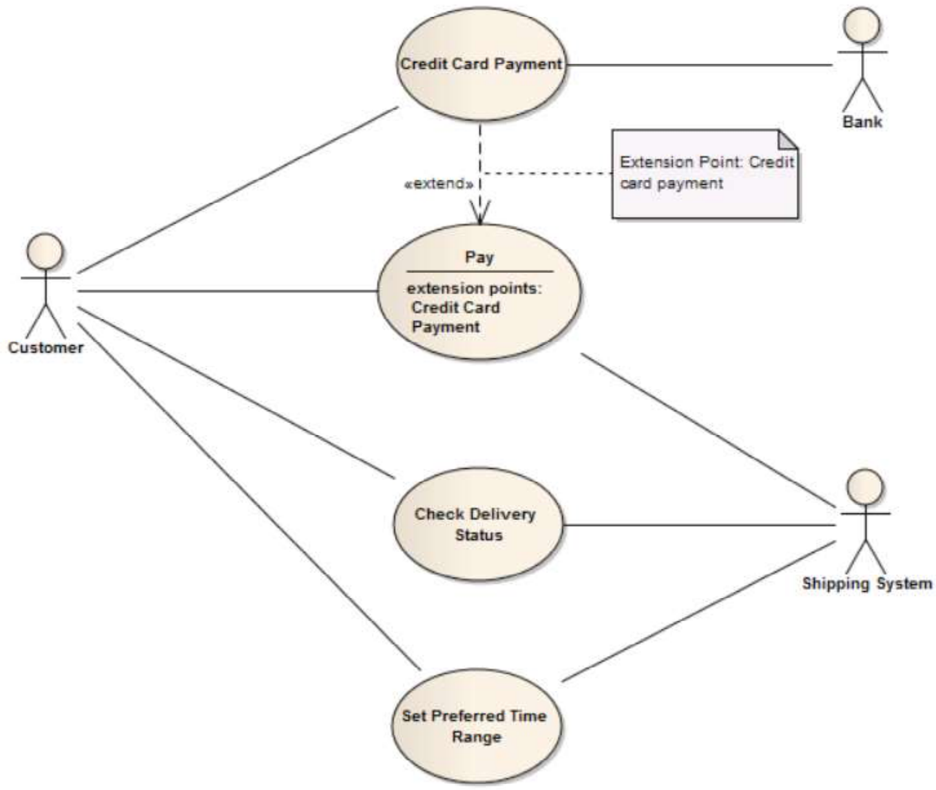

***Fig. 2.2** Use Cases used to Describe the Interactions between a Bank and Shipping System*

In Figure 2.2, we can see that the external systems are represented by actors. The example denotes that payment can be extended whe customer uses to pay via credit card. At the same time, we can also see that not all the payments require a bank's interaction. It completely depends on the method the customer uses. This is known as use case modelling.

Use Case Modelling

- Use cases was originally developed to provide support for requirements induction and currently it is fused into UML.
- Normally, a use case is used to characterize a distinct activity that includes external interaction with a specified system.
- Actors in a use case might be users or any other systems.
- Use case is embodied graphically to present an outline of the use case and in a more comprehensive word-based format.

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Fig. 2.3 *Use Case for Transferring Medical Records*

The Figure 2.3 depicts a use case for transferring medical records of a patient into the patient record system.

Sequence Diagrams

- Sequence diagrams is a portion of UML.
- It is used to determine the various interactions between the actors and the various objects within a system.
- A sequence diagram displays the order of the various interactions that are taking place at the time of a specific use case or instance.
- The objects and actors involved in the interaction are itemized on the uppermost part of the diagram, with a dotted line drawn vertically from them.
- Interactions in between the objects are denoted by annotated arrows.

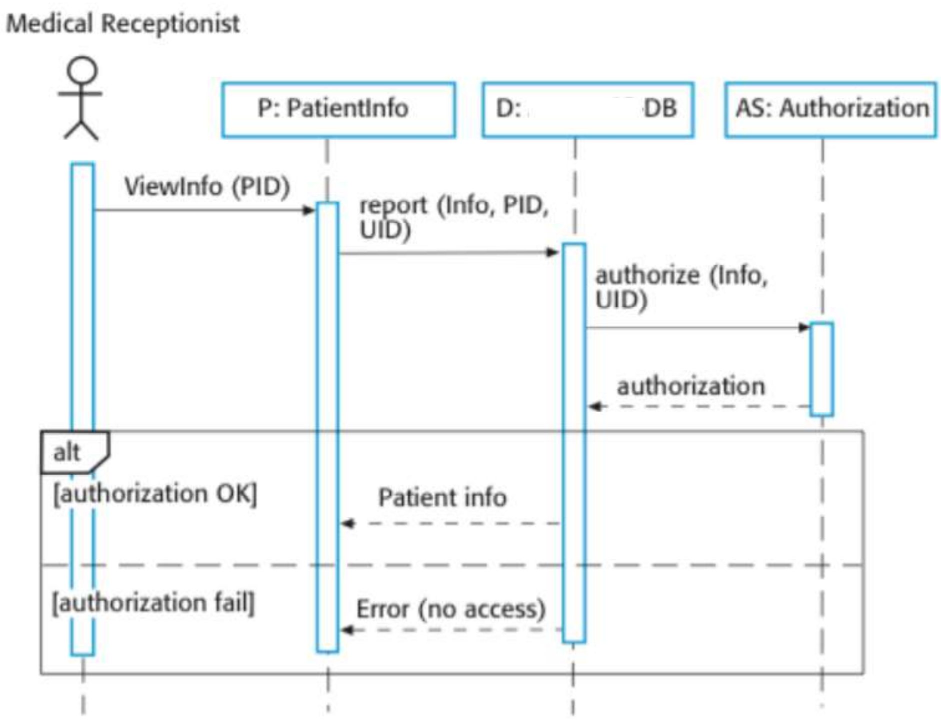

Fig. 2.4 *Sequence Diagram for View Patient Information*

The Figure 2.4 denotes a Sequence diagram for view patient information. We can see that the actor, i.e., the Medical Receptionist is represented at top.

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Structural Models

- Structural models portray the establishment of a system in terms of the various components that the system consists of along with their relationships.
- Structural models may be static or dynamic in nature. Static models demonstrate the structure of the design of the system. On the other hand, the dynamic model depicts the organization of the system during execution.
- A structural model of a system is created during conversing and designing phase of the system architecture.
- "Domains" in this diagram denotes various subject matters that is required to comprehend in building up a system. The domain acts according to its rules and policies.

Class Diagrams

- Class diagrams are normally used or referred to during the development of an object-oriented system model to demonstrate the various classes within the system and the relations between the corresponding classes.
- An object class can be defined as a general definition of an object within a system.
- An association is defined as a linkage between the classes which specifies that there is/are certain relationship in between the designated classes.

Fig. 2.5 *Relationship (1:1 basis) between the UML Class and its Association*

The Figure 2.5 denotes the relationship (1:1 basis) between the UML class and its association.

Behavioural Models

- Behavioural models depict the dynamic behaviour of a system. This is dynamic as the system is getting executed. They demonstrate the happens or what is to be happening when a system replies to an impetus from the environment.
- These impetus or stimuli can be of two types:
  - o Data – Certain data comes to the system which needs to be processed.
  - o Events – Certain event occurs which initiates the processing of the system. Events may consist of allied data at a given point of time. It is not necessary that the event should always consists of data.

Data-Driven Modelling

- Data-driven models portrays the order of actions which is involved during the processing of input data and producing the corresponding output.

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- This is extremely beneficial during the analysis phase as this model demonstrates the end-to-end processing within a defined system.
- It demonstrates the complete sequence of action from input data to output data.
- This model demonstrates the response of the system to a specific input.

Event-Driven Modelling

- Event-driven modelling portrays how a system replies to an external or internal events which is also known as a "Stimuli".
- It this model, it is assumed that a system has a predetermined or limited quantity of states
- It is also assumed that an event (stimulus) might initiate a changeover or transformation from one state to another state.
- Real-time systems ideally are always event-driven system, with negligible processing of data.
- This type of modelling has two states:
  - o **Initial State** – In the diagram, the initial state is defined by a filled black circle. It may also be labelled with specific a name.
  - o **Final State** • In the diagram, the final state is defined by a circle with a dot inside. It may also be labelled with specific a name.

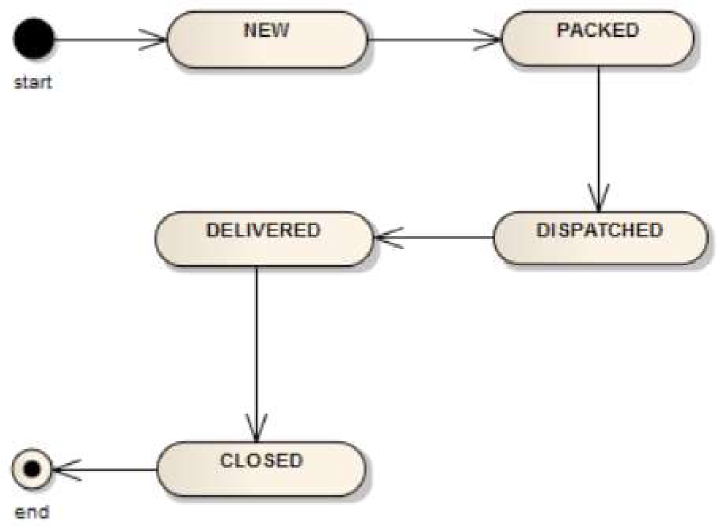

Fig. 2.6 Initial and Final State

State Machine Models

- This model captures the behaviour of the system in terms of responding to external and internal events.
- This model portrays the system's responses to external as well as internal stimuli.

- The model portrays system states as nodes and events as arcs in between the specific nodes. Refer Figure 2.7 in this regard. When an event takes place, the system gets transferred from one state to another.
- "Statecharts" are an essential portion of the UML and are referred to represent the state machine models.

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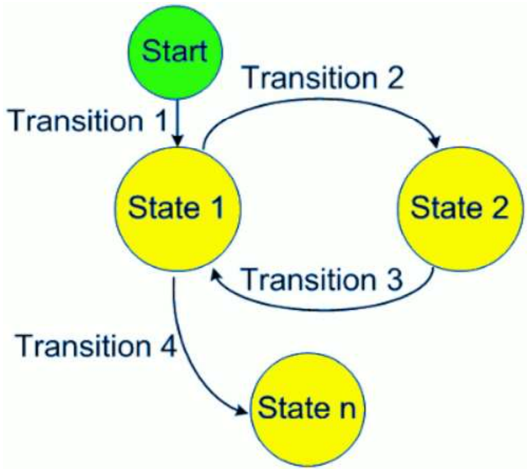

***Fig. 2.7** System States as Nodes and Events as Arcs*

Model-Driven Engineering

- Model-Driven Engineering (MDE) is a method to software development.
- The models are preferred to rather than programs.
- From this model itself the developed programs are execute on a specific platform.
- Model-driven engineering is still at an initial phase of development.

Pros

- Permits systems to be taken into account at an advanced level of construct.

Cons

- It is considered to be the model for abstraction.
- It is not necessarily the correct model for implementation.

Check Your Progress

1. What is system modelling?
2. What are the different types of diagrams used in UML?
3. Name the various kind of models in UML.
4. Define use case modelling.
5. What do mean by structural models?
6. How will you define the Model-Driven Engineering (MDE)?

2.3 SYSTEM SIMULATION

NOTES

Simulation is described as the imitation of the procedure of a real-life process or system for a given point of time. It is the procedure of investigating with a model of the process or system under analysis and it deals with the model of the procedure or system apart from than the original process or system itself. In the field of engineering, simulation is expressed as the procedure or method of investigating with a model of a specific system under research or survey by means of computer related applications or programming.

A model as already discussed, is an explanation of a specific system by symbolic linguistic or concept to be observed as a system using which the environment of objects can be articulated. Thus, a model is an understanding or comprehension of a true system, theoretically.

Generating or emerging a simulation model enables the understanding of the conduct of a process or system as it progresses over a specific time. The simulation model defines the operational procedure of a specific system by means of the discrete and distinct actions and events within the specific system. The interrelation among the associated elements are also constructed within the model. The model then permits the computing device to seize the consequence of the element's events on each other as a dynamic process. A simulation model is referred to examine a variability of "what-if" questions regarding the real identified process or system. The influence or effect of the probable alterations on the system functioning can thus be projected, substitute system configurations can be equated and forecasts regarding systems that are under the preview, deployment or fine-tuning related activities can be accomplished.

In order to accomplish a simulation experimentation, the simulation analyst should take into consideration a holistic view of direction for emerging the simulation model. The most predominant holistic views are the event-scheduling holistic view, the process-interaction holistic view and the activity-scanning holistic view. While dealing with the event-scheduling method, a simulation analyst should focus on the events and their corresponding consequences on the state of the system.

In simple words, a simulation, is just the execution of a specific model. This necessitates the supplementary explanation and characterization of the preliminary conditions of the specific system which is taken into consideration and quantified denominations of the required parameters. Moreover, this involves thorough proficiency on the system we well. For example, for a coffee-shop-system, one has to identify a couple of things beforehand. These factors may be:

- At what time of the day the modelling should be initiated?
- During this time of initiation, how many customers are in the shop and how many employees are on board?
- During this particular time what is the amount of money the cashier has in hand?
- What are the prices for various types of available coffee?
- In that shop, only coffee is served or packaged coffee beans are also sold?

- Are there any food items sold in the coffee shop?
- If yes, then what are the prices of the food items?
- Is there any take-away facility? If yes, then what is the tax implications for those products?
- What are the inflow and outflow rates of customers?

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The modeller should consider all the probable factors, establish the scenario and then conclude with some rules. These may be made simple in the beginning but might become complex at the end. Based on all the findings, these rules should be fit in the simulator for a feasible output. Outputs can be more than one and the simulator has to decide on the best feasible solution.

Why Simulation?

The benefits of simulation are as follows:

a. Establishing a Risk-Free Environment

Simulation modelling establishes a safe and structured approach to assess and discover various "what-if" situations. The consequence of modifying employee levels in a factory may be simulated without making production at jeopardy. Simulation helps us to take the correct conclusion prior to establishing changes in the production environment.

b. Saving time and cash

Simulated experimentations with simulation models are to a lesser extent pricey and consumes lesser time than experimenting with tangible possessions. For example, marketing operations can be simulated without forewarning the opposition or needlessly spending cash.

c. Picturing or Visualization

With the advent of modern technology, simulation models can be animated in 2D or 3D format. This process will certain help permitting various notions and thoughts to be more simply and effortlessly substantiated, corresponded, and comprehended. Analysts and engineers acquire conviction in a model by considering it in act and can evidently establish conclusions to leadership team or senior management.

d. Understanding the Dynamics

In spreadsheet there is a feature known as solver-based analytics. Similar to this, a simulation modelling brings into being the performance of system behaviour over a specific period of time, at any greater details. For example, using simulation, we can check the utilization of a warehouse storage space at any specific date.

e. Accuracy to the Fullest Extent

A simulation model can provide a detailed insight than any analytical model. A simulation model certainly provides improved accurateness and more accurate predictions. Organizations related to mining normally uses simulation model to cut down significant costs by optimizing the usage of assets and knowing their upcoming apparatus requirements.

f. Handle Uncertainty Efficiently

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Every business needs to handle uncertainty. Simulation models can represent uncertainty easily during operation times. At the same it can also predict on the various outcomes during uncertainty. Thus, using this type of model, associated risks can be quantified and for more robust precautions can be taken. In the area of logistics, a representative figure can be generated using simulation, taking into consideration the unpredictable data, i.e., shipment during lead times.

History of Simulation

The historical perspective of simulation is as enumerated in a chronological order.

- 1940 " A technique named 'Monte Carlo' was generated by John von Neumann, Stanislaw Ulan, Edward Teller, Herman Kahn and other physicists researching on a Manhattan project to research on the scattering of neutron.
- 1960 " Initial special-purpose simulation languages were invented, e.g., SIMSCRIPT by Harry Markowitz at the RAND Corporation.
- 1970 " During this time frame, research work was going which was based on mathematical foundations of simulation.
- 1980 " During this time frame, Computer-based simulation application, GUI and Object-Oriented Programming System (OOPS) concept were invented.
- 1990 " During this time frame, web-based simulation, decorative animated graphics, simulation-based optimization, Markov-chain Monte Carlo methods were invented.

Case Study - Identifying the optimum number of employees to carry out a outlined excellence of service to customers visiting the bank.

First of all, for a specific bank, the level of service was well-defined as the normal queue extent. Appropriate structured procedures were then nominated to establish the strictures of the simulation model i.e., the quantity and occurrence of arrival of the customers in the bank, the amount of time a teller takes to complete the operation with a customer and the natural variations which might occur within all of these activities, in specific, rushes during the lunch hour and various complicated requests from the customer.

In this regard, to organize and processes the various bank departments, a flowchart was drafted. In ideal case, a simulation models requires to consider only those conditions which creates a negative impact during difficulties. For example, the availability of back-office employee for do not directly have any relation to the front-ending of the customers or account holders. Hence back-office department is physically and functionally separate from the primary operational activities.

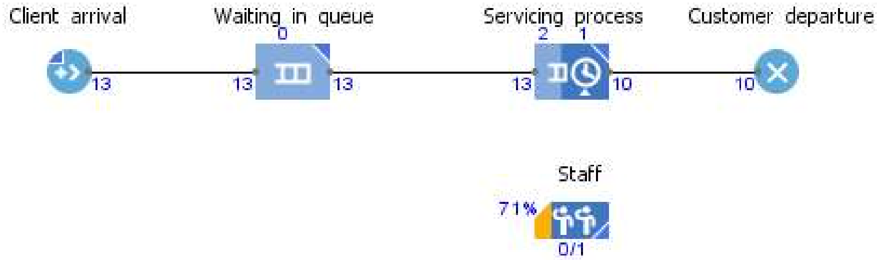

***Fig. 2.8** Flowchart Corresponding to the Structure and Processes of the Department*

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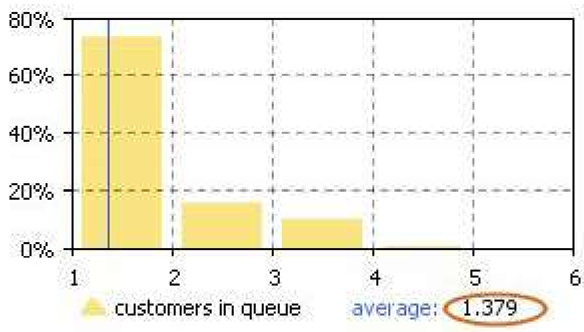
Finally, the relevant data was fed into the simulation model. Post execution of the simulation, refined analysis of the results was acquired. To conclude, if the average customer queue size went beyond the quantified limit, the number of available banking employees should be increased and accordingly a new experimentation was performed.

Time	Event
09:53	customer 7 has been serviced
09:53	customer 9 is being serviced
09:56	customer 8 has been serviced
09:56	customer 10 is being serviced



To obtain an optimal solution, considering various scenarios, with varying parameters, multiple solutions can be generated. The bank management can choose the optimum solution and then implement the same, which is bound to yield better result. The results of the modelling and simulation, therefore, delivers self-assurance and lucidity for analysts, engineers and managers at the same time.

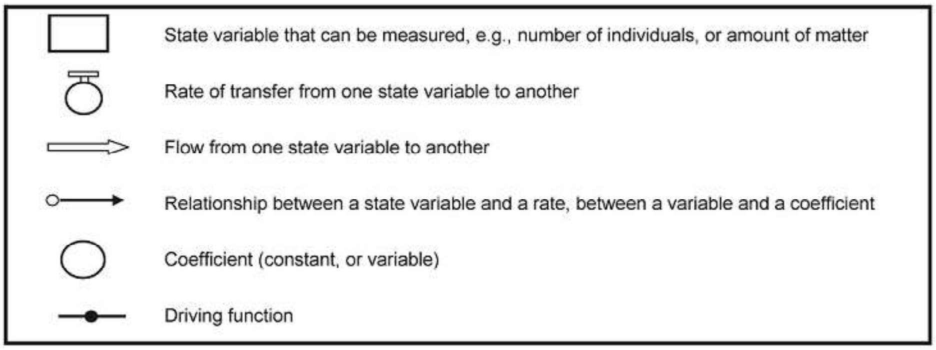


***Fig. 2.9** List of Symbols for Simulation Modelling*

2.4 SIMULATION AND MODELLING PROCESS

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Simulation models comprise of the following components:

- System entities
- Input variables
- Performance measures
- Functional relationships

Following steps are followed during developing a simulation model.

- Step 1 – Problems are to be identified for an existing system or new requirements are to be set for a freshly proposed system.
- Step 2 – Consider the existing system factors and its corresponding limitations which drafting the problem.
- Step 3 – Once the problem area is drafted, data should be collected and processing of the system data should be initiated. During this time a close observing should be done in terms of the system's performance and overall generated result.
- Step 4 " Development of the model should be based on the network diagrams and should be verified using numerous verifications methods.
- Step 5 – Validation of the model to be performed by associating its performance against numerous conditions against the system in production.
- Step 6 – For future use, a document should be drafted for the model, It should include, in details, the objectives, assumptions, input variables and performance.
- Step 7 – As per requirement, selection of a appropriate experimental design should be accomplished.
- Step 8 – Several appropriate experimental conditions should be induced on the model and the result should be observed and noted.

Performing Simulation Analysis

Following steps should be performed during the simulation analysis.

- Step 1 – The problem statement should be drafted.
- Step 2 – Input variables should be chosen and entities should be created for the process to simulate. There should be 2 kinds of variables decision variables and uncontrollable variables. Decision variables should be controlled by the programmer and the uncontrollable variables should be the random variables.
- Step 3 – Constraints should be created based on the decision variables via assigning it against the process for simulation.
- Step 4 – Output variables should be determined as well.
- Step 5 – Real-life data should be collected from the system. This should go as a input into the simulation.

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- Step 6 – A flowchart should be developed which should track the progress of the simulation process.
- Step 7 – A simulation software should be chosen as appropriate to execute the created model.
- Step 8 – The simulation model should be verified by comparing its generated result against the system in production.
- Step 9 – An experiment should be conducted on the model by altering the variable values to identify the optimum solution.
- Step 10 – To conclude, these outcomes are to be applied into the production system. Production environment is the real environment.

Advantages of Modelling & Simulation

Following are the advantages of using Modelling and Simulation:

- Ease of understanding – It becomes easy to understand how the real system works and operates without going into the details of the real-time systems.
- Testing can be done easily – It is easier to tweak the values and variables in the simulated environment without going into the details of the real-time systems.
- Upgrade can be done easily – It is easier to apply various changes by applying various configurations and conditions.
- Constraints can be easily identified – Bottlenecks, which are responsible to causing delays in the system, can be analysed and subsequent solutions can be concluded to eliminate the root cause of the problem.
- Problems can be easily identified – It is easier to identify the problem area and convert the work around into a known error easily. This can be accomplished easily as the modelling and simulation are able to understand all the interactions and at the same time able to analyse problem easily. With this, new processes, policies, operations and procedures can be identified without creating an effect on the real system.

Disadvantages of Modelling & Simulation

There are certain drawbacks of using Modelling and Simulation. They are:

- Designing a model is considered to be an art. It certainly requires huge amount of domain knowledge, sufficient training and adequate experience. Subject matter experts can only design a successful model.
- Several operations are accomplished on the system model by using random number. With random numbers, there is a variation in the predicting result.
- Developing successful and effective involves sufficient man-hours. It is a time-consuming procedure.
- It is difficult to translate the results produced by the simulators. It can be accomplished by experts only.
- Developing and executing simulation processes are pricey.

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Application Areas of Modelling & Simulation

Modelling & Simulation are applicable on various areas. They are:

- Applications related to defence
- Training and support
- Designing of semiconductors
- Applications related to telecommunications
- Field of civil engineering designs and its corresponding presentations
- Business models related to E-business
- Internal structure of a complex biological system
- Routing algorithms
- Assembly line
- Testing of new designs
- Testing of policies
- Verification of analytic solutions

Components of Modelling & Simulation

Following are the components of Modelling & Simulation:

- **Object** is considered to be an entity which certainly survives in the real world and is used to study the behaviour of a specific model.
- **Base Model** is a theoretical description of object properties and its corresponding behaviour. It is constant and valid across the entire model.
- **System** is the coherent object within a certain condition. It certainly exists in the real world.
- **Experimental Frame** is used to learn a system in the production environment. It consists of various investigational conditions, characteristics, purposes, etc. The basic experimental frame comprises of two sets of variables. They are:
    - o Frame Input Variables
    - o Frame Output Variables

The Frame input variable is accountable for corresponding the inputs applied to the real-world system or a specific model.

The Frame output variable is accountable for corresponding the output values to the real-world system or a specific model.

- **Lumped Model** is a precise description of a system and it maintains the quantified conditions of a specified Experimental Frame.
- **Verification** is the method of comparing two or more items to guarantee their correctness. In Modelling & Simulation, verification can be performed by associating the reliability of a simulation program and the lumped model to guarantee their throughput. There are numerous conducts to accomplish endorsement procedure.

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- **Validation** is the method of comparing two consequences. In Modelling & Simulation, validation is accomplished by comparison of experimentation measurements with the simulation consequences within the framework of a specific Experimental Frame. The model is considered to be inacceptable, if the results do not match.

System State Variables

The system state variables are an array of data which is needed to describe or express the internal process inside the system at a certain point of time.

- In a **discrete-event model**, the system state variables continue to be constant over interludes of time and the values alter at demarcated points known as event times.
- In **continuous-event model**, the system state variables are described by differential equation results whose value alters uninterruptedly over time.

Following are the system state variables which are used frequently.

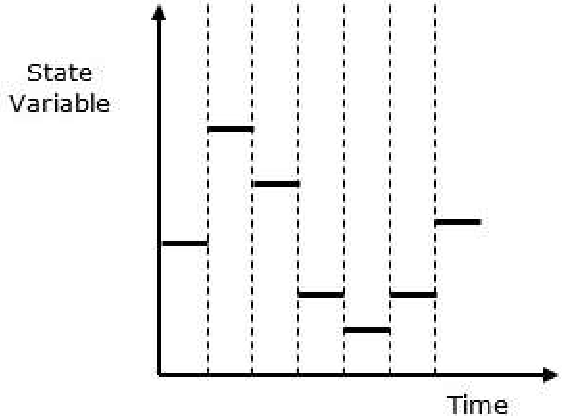
- **Entities & Attributes** –An entity signifies an object. Its value can be static or dynamic in nature. This value is dependent on the process with other entities. Attributes are always local values which are utilized by the entity.
- **Resources** –A resource is an entity which caters service to one or more dynamic entities at a given point of time. The dynamic entity can certainly ask for one or more units of a resource. In case accepted, the entity can use the resource and release once the work is accomplished. In case disallowed, the entity can get connected with a queue.
- **Lists** – It is used to denote the queues which are used by specific entities and resources. There are numerous possibilities of queues such as LIFO, FIFO, etc. and is depended upon the process.
- **Delay** –It is an unlimited timeframe that is produced by some amalgamation of complex system conditions.

Classification of Models

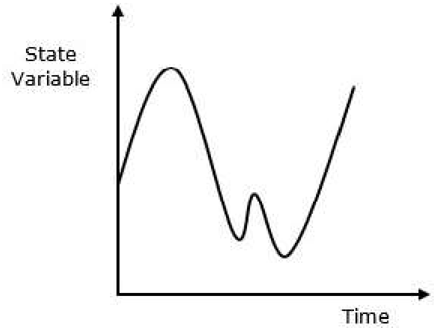
There can be various categories of a system. They are:

- **Discrete-Event Simulation Model** –In this model, the value of the state variable gets altered change only at a given specific points of time where the events take place. Events will solely take place at a demarcated activity time and delays.
- **Stochastic vs. Deterministic Systems** –Stochastic systems are not at all impacted by randomness and its corresponding output is not at all a random variable. Deterministic systems are certainly impacted by randomness and its corresponding output is always a random variable.
- **Static vs. Dynamic Simulation** –Static simulation consists of models which are not at all impacted or influenced with time. As an example, we can consider the monte carlo model. Dynamic simulation consists of models which are impacted with time.

- **Discrete vs. Continuous Systems** –Discrete system is impacted by the state variable modifications at a distinct point of time. Its behaviour is portrayed in the following graphical image.



Continuous system is impacted by the state variable, which gets altered on a continuous basis as a function with time. Its behaviour is represented in the following graphical image.



Modelling Process

The Modelling process constitutes of the following steps as shown in Figure 2.10.

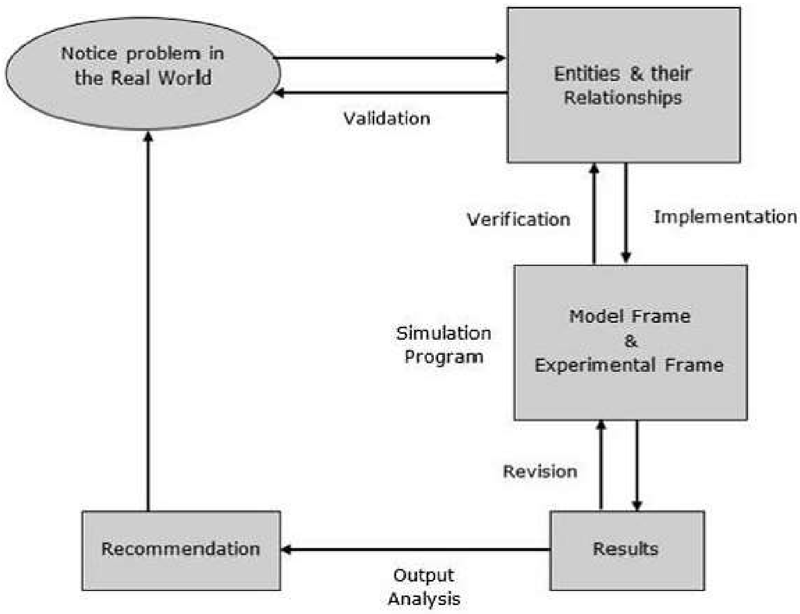


Fig. 2.10 *Modelling Steps*

Step 1 – The problem is examined. In this stage, the problem is understood and proper classification (deterministic or stochastic) of the problem is determined.

Step 2 – The model is designed. In this stage, the following tasks are accomplished to design the model.

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- Data is collected as per the system's behaviours and forthcoming necessities.
- System's features are analysed. Corresponding assumptions and required actions are to be considered so that the model is successful.
- Variable names, associated functions, corresponding units, appropriate relationships, and corresponding applications are determined so that they can be utilized in the model.
- The model should be solved by means of an appropriate and feasible technique. The results are to be verified using appropriate verification methodologies. After this is accomplished, the results are to be validated.
- A report and essential documentation are to be prepared. This should include the outcomes, interpretations, conclusion and further proposals.

Step 3 – With respect to the model, after the completion of the process, it is mandatory to furnish the appropriate feasible recommendations. The feasible recommendations entail of further investment, required resources, suitable algorithms, appropriate feasible techniques, etc.

Validating the model is one of the major challenges that the simulation analyst experiences. The simulation model is considered to be valid only if the created model is a replica representation of the original system. In other case the model is not valid.

For validating a model, validation and verification are the two steps which must be followed. This is a common practice in any simulation project.

- **Validation**: It is the procedure followed to compare the two results. In this process, the comparison is performed between the conceptual model and the real system. In case the comparison yields accurate results, then the model is valid, else the model is not valid.
- **Verification**: It is the process of finding accuracy. Here, the comparison of both the systems produces a result by means of which the accuracy can be judged. In this procedure, the comparison is accomplished between the model's deployment and its accompanying data with the developer's conceptual explanation and conditions.

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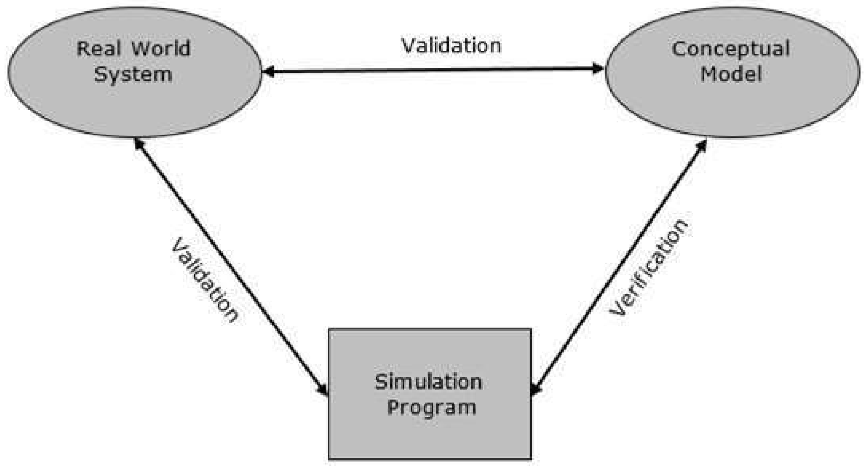

Fig. 2.11 *Relation between Real World and Conceptual World*

Techniques for Verification & Validation

There are numerous methods and practices utilised to accomplish verification & validation of simulation model. Following are some of the familiar methods:

Techniques to Perform during Verification of the Simulation Model

Following are the steps to accomplish the verification of simulation model

- Writing programs and sub programs via the programming skills and debugging the programs wherever necessary.
- More than an individual should read the program. This should be done via the "Structured Walk-through" technique.
- The intermediate results should be tracked and should be compared to with the observed results.
- Using of various input combinations and checking the output from the simulation model.
- Distinguishing the final simulation result with the analytic results.

Techniques to Perform Validation of Simulation Model

Step 1: The model should be designed with high validity. This is achieved via the following steps:

- The design requires expertise. Hence, during the design of the model experts should be consulted.
- The model should be designed in such a way so that it should continuously communicate and interact seamlessly with the clients throughout the process.
- The output should be supervised by a system experts at every given point of time.

Step 2: The model should be tested with assumed data. This should be accomplished via employing the supposition data into the model and further examining it in a quantitative fashion. Complex and delicate analysis can also be accomplished for observing the effect of variation in the outcome when noteworthy modifications are crafted in the input dataset.

Step 3: The illustrative output of the simulation model should be established. This can be attained by the help of the following steps:

- Required to compare and conclude how adjacent the simulation output with the production system output is.
- Turing Test should be used for the comparison related activities this test furnished the data in the system format, which is to be interpreted by experts in this field only.
- Certain statistical methods are also used to compare the model output with the production system output.

Model Data Comparison with Real Data

Post development of the model, comparison of the output data with the production or live system data has to be performed. Two approaches as follows are normally used to perform this comparison related activities.

Validating the Existing System

For this method, Live system inputs of the model is to be compared with its corresponding output with that of the live production inputs of the live production system. This is a validation process which is very straight forward. But sometimes it yields certain hurdles while carried out. These hurdles can be like in case the output needs to be compared to average length, waiting time, idle time, etc. it should be compared by the help of various statistical assessments and hypothetical testing. Some of the frequently used statistical testing methodologies are chi-square test, Kolmogorov-Smirnov test, Cramer-von Mises test, and Moment's test.

Validating the First Time Model

Let us take into account that we need to define a proposed system which is not existing at the present moment. It is completely for a new system. So, there is question of any past data existing for comparison related activities. In such case, it is required to use a hypothetical system based on certain expectations and assumptions. Following are the useful steps which will make the process mode efficient and easier.

- **Subsystem Validity**: It is not necessary that a model should have any existing system to compare it with, but there may be a situation where there might exist a known subsystem. In such case each of the validation should be tested discretely.
- **Internal Validity**: A model which has excessive amount of internal discrepancy shall be immediately disallowed as a stochastic system. With extreme variance, its associated internal processes will conceal the modifications in the output as there exists a huge number of changes in its inputs.
- **Sensitivity Analysis**: This is an analysis which delivers information about sensitive parameters for a specific system. In such case it is mandatory to pay advanced consideration.

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- **Face Validity**: In case a model performs on reverse logics, then this model should be immediately disallowed even if the model is behaving similar to the production system.

Various steps followed in a Simulation Study

1. Problem formulation: This is the statement of the problem. It should be clear, concise, and understandable. This has to be very clear as based on this identified problem statement, the solution is to be identified.
2. Setting of objectives and overall project plan: The activity objectives and overall project plan should be established. The roles should be defined to all individuals involved in the project. The communication line should be established. The project progress template should be set. The project stakeholders should agree and sign-off this objective and plan. The scope of the model and the alternatives should also be captured in this phase.
3. Model conceptualization: There are various categories of models. The right and appropriate model needs to be chosen. This is the work of an expert and hence the subject matter expert in this area should be involved.
4. Data collection: Appropriate data needs to be collected for testing purpose. The data should be relevant, complete and in correct format. Incorrect or inconsistent data might lead to incorrect outcome of the results.
5. Model translation
6. Verified? : The programs are verified with test data and the correctness of the execution is checked. In case not, the associated programs are debugged so that the correct logic is in place to generate the correct desired data.
7. Validated? : In this phase it is checked whether the model precisely characterize the real system at the time of operation with real data?
8. Experimental design: Feasible alternatives are identified and simulated in this phase.
9. Production runs and analysis: In this phase, execution of alternatives and meaningful solutions are performed and the outcome is compared with the desired result.
10. More runs? : In this phase, it is to be decided whether mode runs or additional experiments are required or sufficient recommendations and feasible inferences are gathered.
11. Documentation and reporting: Standard program documentation, Progress documentation and Reports are the 3 documentation which needs to be developed and signed off as appropriate. In case training is required, the same needs to be catered to the relevant individuals as appropriate.
12. Implementation: This is the last phase of the simulation study. With the conclusions the identified solution shall be implemented in the production environment with appropriate and relevant approvals from the senior management.

The steps are given below in a flowchart (see Figure 2.12) format.

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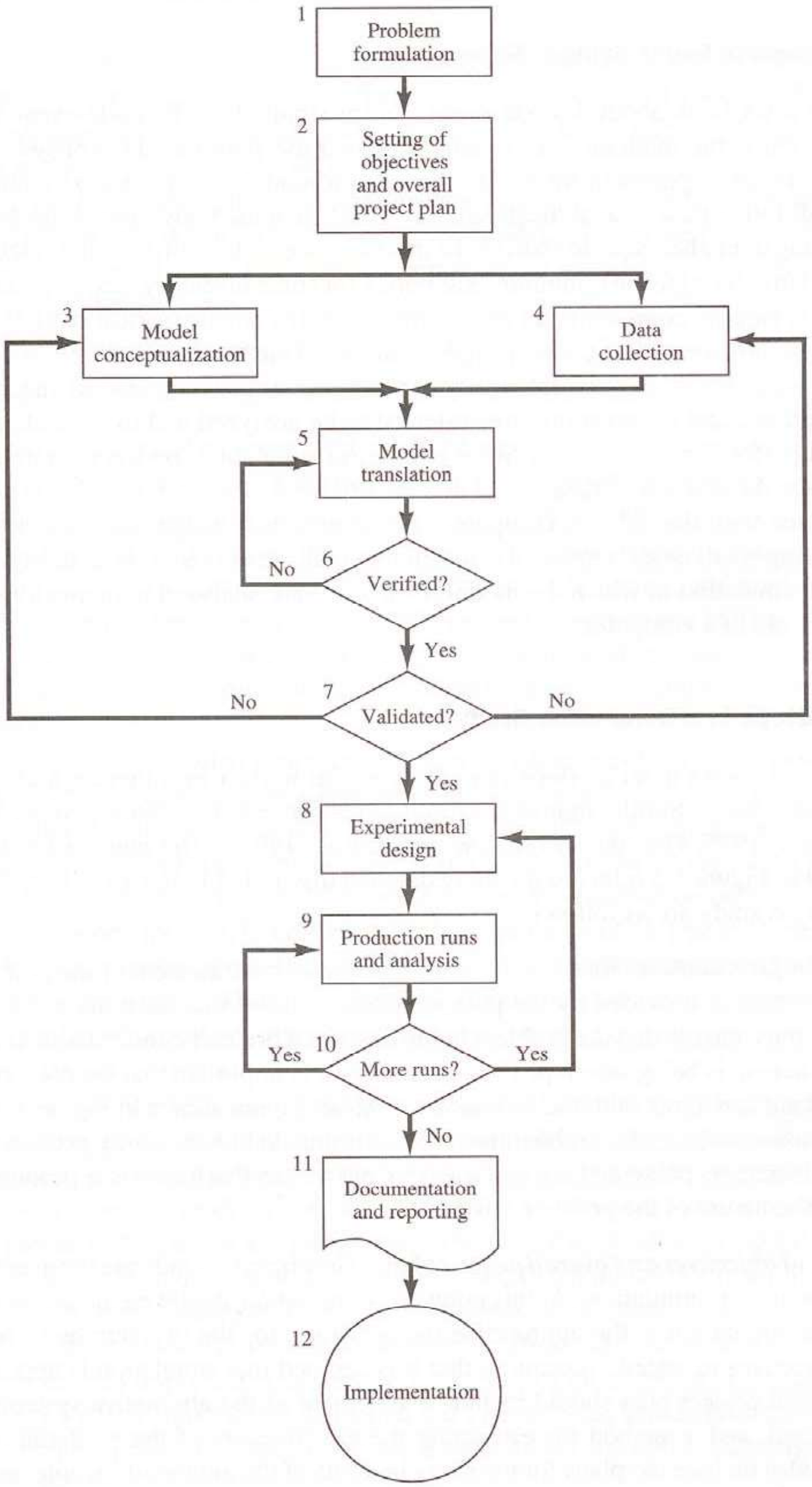

Fig. 2.12 *Steps followed in a Simulation Study*

System modelling and simulation is used for the following purpose:

- Applying of scientific discerning to the scrutinize multifaceted systems and associated processes.
- Understand significant perceptions in various types of modelling and simulation.
- Study in the area of ambiguity and arbitrariness of models by the help of statistical distributions.
- Establishing of a proposition and simulate it for testing purposes.
- Collecting of sample data, identify associated faults after processing the collected sample data and further perform require analysis with the simulation outputs.
- Comprehend the process of generating various kind out outputs and variates.
- Understanding of the scope of application and associated shortfalls of a system.
- Incorporate statistical methods and approaches to for scientific definitions and suppositions.
- Fabricate, authenticate and confirm system and its associated processes.

Examples

- **Input Data Modelling**

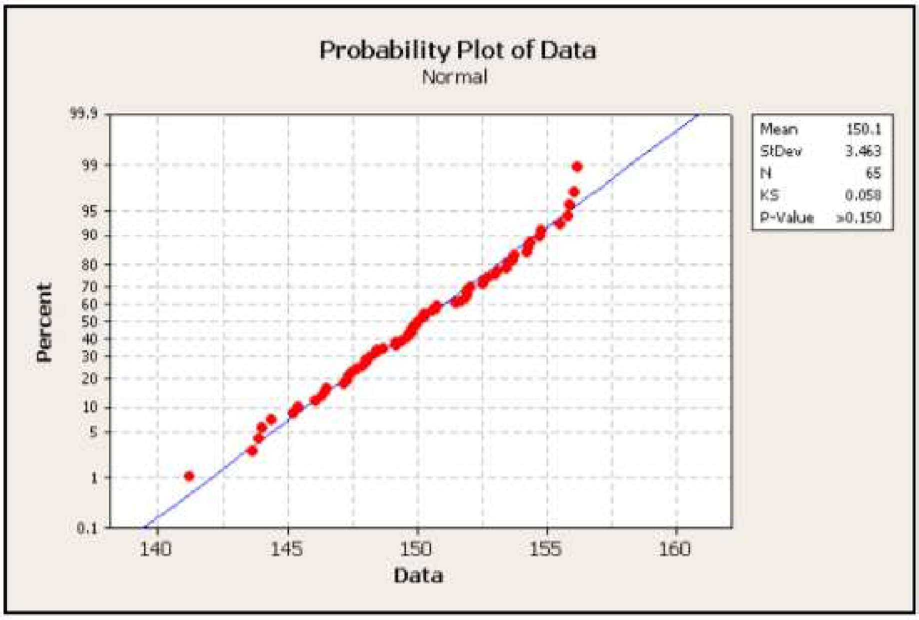

Fig. 2.13 *Q-Q Plot for Testing Normality of Data*

Figure 2.13 with appropriate distributions for input data and it is considered to be a is a major simulation activity in the real world. Defective models of the inputs will generate faulty explanation and will provide deceptive inference (also known as "Garbage-In-Garbage-Out").

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Following are the four steps which are followed to develop a beneficial model of input data:

(a) Collection of relevant data from the production or real system under consideration.

(b) Recognizing of a appropriate probability distribution for representing the associated input process.

(c) Selecting relevant parameters that identifies a precise instance of the distribution species

(d) Determining the identified distribution and the related parameters for appropriate fit.

- **Verification & Validation**

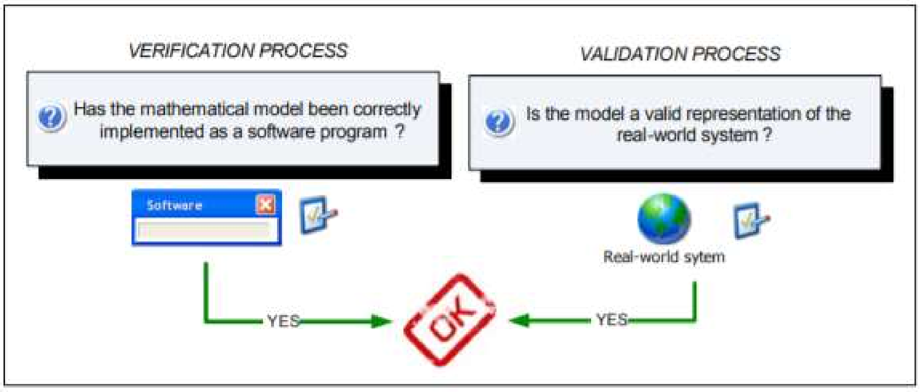

Fig. 2.14 Important Factors for Verification and Validation Processes

Verification is associated with building the model appropriately. **Validation** is related with building the accurate model. This is the basic difference between verification and validation.

The main aim the validation process are as follows:

(a) To yield a model that signifies real time system behaviour i.e., the model would be meticulously sufficient enough to be used as a auxiliary model for the real life system and this is for the purpose of studying the system closely.

(b) To find the acceptable of the model in such a level so that it can be used by the appropriate individuals for decision making

Verification and validation of simulation models should have immense reliability because vital decisions are based out of this model. This is the reason, accurateness of the result generated via this model should never be the theme to query or further examination.

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• Output Analysis

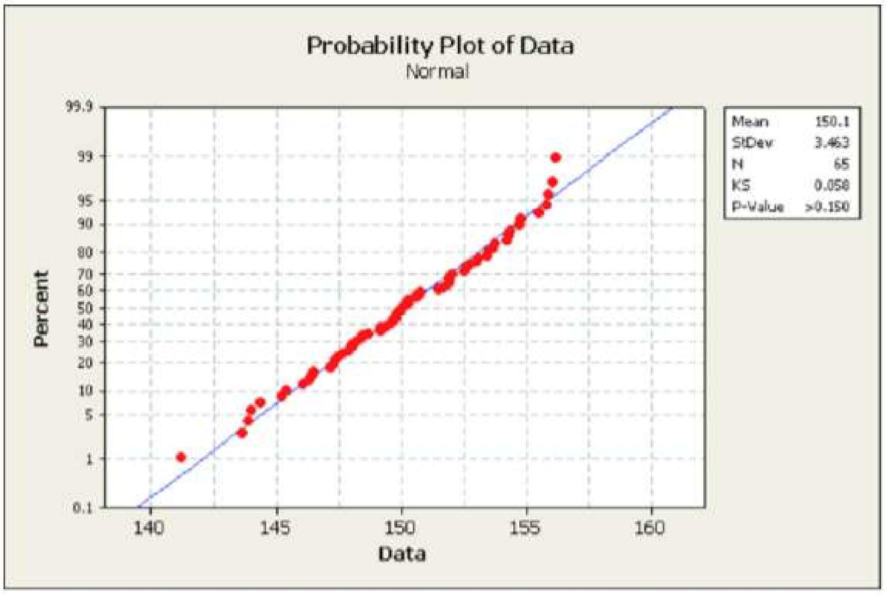

Fig. 2.15 *Graph of System Performance under Various Scenarios*

Output analysis is the examination of data generated by a simulation. Its purpose is either to predict the performance of a system or to compare the performance of two or more alternative system designs.

The need of statistical output analysis is based on the observation that the output data from a simulation exhibits random variability when random number generators are used to produce the values of the input variables.

Several variance-reduction techniques can be employed to reduce the number of replications that are necessary to reach a certain value for the estimation accuracy.

Evaluation of Alternate Designs

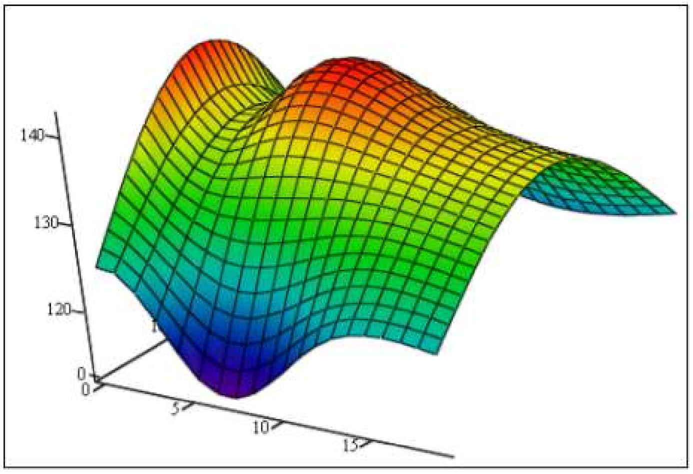

Fig.2.16 *A Response Surface for Optimal Design*

One of the greatest importance of using simulation is to compare the design of alternative systems.

An applied approach to differentiate multiple system designs is to calculate the points and interval estimates of the difference, in mean performance of the parameters of an explicit system noteworthy variance between both designs (of systems) will occur in case the resulting confidence interval will contain no 0 value.

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How Simulation Works

Simulations work by developing models or systems to recreate real-world scenarios. A model refers to any setup, device, or representation used to describe or simulate a process when it cannot be experienced directly. The steps needed to conduct a simulation are as follows:

1. Scenarios or situations requires to be identified. Need to judge whether it is a new model or recreation of an existing model. Along with this the feasibility has to be assessed.
2. The main purpose or objective of the simulation has to be established and need to ascertain that the planned simulation will cater the desired outcome.
3. To start off, assembling of all the input values related to the tool, parts of the machine, or models is a mandatory. The execution methodology of the model needs to be identified. Testing of the outputs is a mandatory activity. Testing can be done either in short groups or a complete test can also be done. Testing should ensure that the model is working perfectly in terms of its functionality. Identified issues should be addressed prior to the simulation is performed on a huge level.
4. For instance, a simulation is performed to make a readiness during a crucial to the outcome. Individuals involved in performing the modelling and simulation should have adequate knowledge in this area. The simulation test should consist of all probable tools and machineries required during a live situation. The simulation run should also consider safety measures as well. Speed of achieving a recovery in any situation should also be simulated. The turnaround time should be brought down with the help of simulation.

Processes which can be Simulated

Diversity of processes which can be simulated are as follows:

- Experiments related to science and technology
- Discovery in the area of science and technology
- Drills related to natural catastrophes – e.g., storm, earthquake or a fire drill.
- Forecasting of weather
- Flight trajectory
- Processes related to healthcare i.e., surgery, MRI, etc
- Shooting of films
- Safety measures
- Education & Training

Steps for Developing a Simulation

- Step 1. The problem needs to be identified

NOTES

- Step 2. The problem needs to be formulated.
- Step 3. Real time production data is collected and processed.
- Step 4. Designing and developing of the model.
- Step 5. Accomplishing the validation of the model.
- Step 6. Adequate documentation to be done for future need.
- Step 7. A suitable experimental design needs to be identified.
- Step 8. Experimental conditions require to be set for the execution in the simulation software.
- Step 9. The simulation needs to be accomplished.
- Step 10. Output results are to be interpreted and presented in a presentable manner.
- Step 11. Further course of actions are to be suggested.

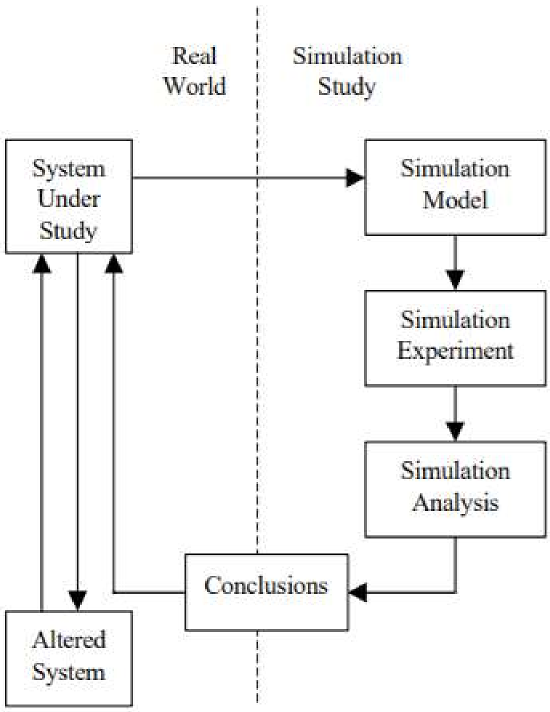

***Fig. 2.17** Simulation Study diagram*

The Figure. 2.17 is a diagram of a simulation study. The repetitive character of the process is specified by the system within the preview of experiment or learning becomes the transformed system which then gets converted into a system which will be within the preview of study and it goes on repeating. In the area of simulation experiment, human's choice and conclusions is always essential at all the phases i.e., development of the model, designing of the research, analysing of output, preparation of inference, and creating conclusions to modify the system under the preview of observation. Execution of the simulation is the only area or phase where human intervention is not at all required. It is performed by the simulation software only. Simulation software is the most important aspect of the entire exercise. Considering the modern technological trend and complexity of the system, Without the software, it is impossible to overcome the complexity of the simulation activity.

What makes a Problem Suitable for Simulation?

NOTES

In normal situation, when a requirement for modelling and analysing arises and there is complex randomness within the system, simulation process is the only tool which can be referred to for better outcome. Circumstances where simulation, modelling and analysis is referred to comprise of the following:

- It is not possible or tremendously high-priced to monitor specific processes in the real world, e.g., cancer related statistics for the next two year, performance of a space shuttle in space, business increase / decrease related consequence of online advertisements for a specific company.
- Several problems where mathematical models can be established for formed but it is not possible to perform a associated and appropriate analytical solutions (e.g., scheduling of staffs in a shop, high order of differential equations) or the process are very much complex (e.g., monitoring of the stock market, huge number of job scheduling, in terms of millions, within a system).
- Validation of mathematical model which is explaining a specific system. This kind of problems are either impossible to perform manually or they are very much expensive for execution for absence of data in either form.

Why a Simulation Fails?

- There are no clearly defined objectives.
- In case a analytic solution is available and still we are using a simulation
- Model has discrepancies.
- The simulation model is very much complex or is very simple.
- Assumptions or considerations are incorrect.
- Assumptions which have insufficient or no conditions defend. This is tremendously significant and it is very much recommended that assumptions which are made to model the simulation should be tested and analysed and documented systematically.
- Using of incorrect input distribution of probability.
- Replacing stochastic by its deterministic which should not at all be the case.
- Using of incorrect measure for obtaining performance.
- The simulation program containing defects or bugs.
- Using of typical statistical method or formulas which considers independence in analysing the output of the simulation.
- Preliminary preference in the simulation output data.
- Making only a single simulation run for a typical configuration. There should ideally be multiple runs.
- Inadequate scheduling strategies

- Budgeting constraints
- Inadequate communication among the individuals who are included in the study of the simulation activity.

NOTES

To build up a model, in case a structured development is required using computer programming language, a rigorous approach should be in in place. This should ensure the correctness of the simulation software in terms of its functionality. The development team should keep in mind that the software should have the following quality factors and it is a mandatory from all aspects:

- Auditability
- Maintainability
- Consistency
- Modularity
- Correctness
- Portability
- Data commonality
- Dependability
- Efficiency
- Reusability
- Expandability
- Robustness
- Flexibility
- Security
- Integrity
- Testability
- Interoperability
- Usability

Check Your Progress

7. What are benefits of simulation?
8. Write the components of simulation?
9. What are the application areas of simulation?

2.5 INTRODUCTION TO MODELLING AND SIMULATION II

The words model and simulation mean diverse to various domains. Similarly, it means different to various groups of people. Conventionally in the domain of technology the word model is used as a noun and the meaning of it is just a representation or reproduction, of a production system. It is normally a physical entity. The word Simulation, on the other side, has means the act of presenting an instance of something (e.g., process or procedure) without the realism. For example, an explanation of simulation is the performance of using a computer to produce a resolution for a specific dilemma which can't be resolved by conventional mathematics. There is another perspective for the term modelling. The modelling comprises both the building of a model and the corresponding manipulations of the specific model (i.e., the simulations).

Simulation modelling software normally considers the following three worldviews:

- Scheduling of events
- Interaction with processes
- Scanning of associated activities

As per the above mentioned three factors, each point puts its primary focus on the events, the associated processes or the specified activities in a simulation, as appropriate. During dealing with scheduling an event, a model developer should outline the corresponding appropriate model logic and system state changes that should take place at the time of the occurrence of any event.

A process is defined as a series or progression of events, activities and other time delays which are in conjunction with a single entity as it streams within a system. For example, a customer process in a bank comprises of a entrance event (to the bank lobby), ready and standing in a queue (which is considered to be a delay), a facility time by the corresponding teller and finally the service accomplishment event. Conceptually, the service time is considered to be an activity and the teller is considered to be a resource.

Activity scanning is the process which provides a method to describe the model logic by concentrating on various activities involved in accomplishing the complete process

A simulation model is considered to be a descriptive model of a process or system, and normally consists of parameters which permits the model to be configurable.

As a descriptive model, the simulation model can be used for experimentation purposes, can be used for evaluation and comparison purpose to identify various alternatives. Evaluation, comparison and analysis are the crucial motives for performing simulation. Forecast of system throughputs and identifying the troubles of a system along with probable causes are the primary output results.

When handling a simulation study via its numerous phases, a decent and experienced manager should must closely consider the following guidelines and have a close eye on the following probable drawbacks.

- The simulation project should have distinctly demarcated and attainable goals. The goals should be kept in mind during the execution of the entire project life cycle.
- The simulation project should be assigned with acceptable resources. These resources should be subject matter expert and should possess adequate experience.
- The most important part of the success involves to get confidence from the higher management. Their involvement in the project is a mandatory.

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- It is essential to have periodic review meetings with all possible stakeholders of the project. The communication should be kept opened.
- Any questions coming in mind should be asked. Confirmation is always better than assumption.
- Concept of assumption should be excluded. Prior to do something, things should be confirmed.
- Common understanding on project scope and project goals should be developed.
- All assumptions should be documented.
- All changes to assumptions should also be documented.

Following are the primary pitfalls and grounds of simulation project getting unsuccessful. This includes:

- Creep in the scope
- Slippage in time
- Overrunning of the project
- The project deals with needless, redundant and superfluous particulars which is not at all required in the project.
- Incorrect or inappropriate skill sets
- Major stakeholders of the project getting involved in the project for the first time in the final presentation and declares that the solution has lots of issues.

Prior to conducting simulation related experiments, it is mandatory for an analyst to decide on the following:

- The input parameters for the test should be diverse, the range of the input parameters along with its acceptable and justifiable amalgamations
- Runlength of the model (For how long the simulation should run)
- To decide on the model warm-up period for steady-state analyses
- The amount of expected statistical replications.

During the initiation phase of performing the simulation, following are the three most important considerations which should be kept in mind. They are:

- The scope and the boundary of the model
- Detailing levels
- Scope of the project.
- A simulation of a system is defined as the process of a model which is a depiction of the system. Example of systems can be:
- Telecom system (continuous flow of message traffic)
- Road traffic system
- Factory
- Manufacturing unit
- Economy
- Airport/Port

- Behaviour of an operating system

The model is agreeable to change that would be dreadful, very much costly or unreasonable to accomplish on the system it depicts.

With the help of experimental activities with the operation of the model, possessions of the real live system can be concluded, offering understanding and clarifications for system behaviour and prognostic competences of system behaviour under identified functioning parameters.

It is beneficial to pursue to categorize models. Models can be categorized as follows:

- A **scale model** is a bodily depiction of lesser magnitude, often with only indispensable particulars (e.g., map of a country or a model of a ship).
- A **mockup** is a full-scale depiction, typically overlooking unseen aspect (e.g., interior of a ship cabin, an aircraft control panel).
- An **analog** forecast the behaviour of an analogous system.
- **Symbolic models** are the abstract illustration for the system under the purview. A symbolic model can also be intuitive as well or can also be logical.

We can additionally describe a symbolic, logical model using five characteristics. They are defined in the following figure 2.18.

***Fig. 2.18** Characteristics of Symbol Model*

2.6 DISCRETE SYSTEM MODELS

NOTES

In discrete systems, the modifications in the system state are intermittent and every change happening within the state of the system is known as an **event**. The model which is used in a discrete system simulation normally consists a set of numbers to characterize the situation of the system and is known as the **state descriptor**.

Following is the graphical illustration (Fig. 2.19) of the behaviour of a discrete system simulation.

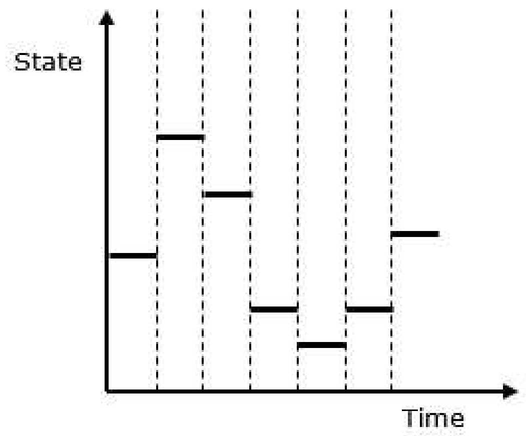

Fig. 2.19 *Behaviour of a Discrete System Simulation*

Key Features of Discrete Event Simulation

Discrete event simulation is usually performed by a software considered in high level programming languages, e.g., Pascal, C++, or any other specialized simulation language.

Following are the five main characteristics:

- **Entities** – They are the real elements of a system e.g., machines parts.
- **Relationships** – It establishes the relation between two entities.
- **Simulation Executive** – It is accountable for governing the progress time and execution of discrete events.
- **Random Number Generator** – It assists to simulate various data which is flowing into the simulation model.
- **Results & Statistics** – It confirms the model and delivers its throughput procedures.

Time Graph Representation

Every system is dependent on a time parameter. In a graphical representation it is denoted as clock time or time counter and primarily it is set to zero. The time is updated on the basis of the following two factors:

- **Time Slicing** – It is the time which is demarcated by a model for apiece event till the nonappearance of any event.

- **Next Event** – It is the event which is demarcated by the model for the subsequent event to be executed in place of a time interval. It is definitely very much efficient with respect to Time Slicing.

NOTES

Simulation of a Queuing System

A queue is the amalgamation of each and every entity in the system which is being already attended and those in queue for its chance.

Parameters Used

The list of parameters used in the queuing system are as follows:

Symbol	Description
λ	Signifies the arrival rate i.e., the number of arrivals per second
Ts	Signifies the mean service time for every arrival. It excludes the waiting time in the queue
σTs	Signifies the standard deviation of the associated service time
ρ	Signifies the utilization of server time, both at the time of idle and busy
u	Signifies the intensity of the traffic
r	Signifies the mean of entities within the system
R	Signifies the overall number of entities within the system
Tr	Signifies the mean time of an entity within the system
TR	Signifies the overall time of an entity within the system
σr	Signifies the standard deviation of r
σTr	Signifies the standard deviation of Tr
w	Signifies the mean number of entities coming up in the queue
σw	Signifies the standard deviation of w
Tw	Signifies the mean waiting time of all entities
Td	Signifies the mean waiting time of the entities coming up in the queue
N	Signifies the number of servers within a system
mx(y)	Signifies the y^{th} percentile which defines the value of y below which x occurs y % of the time

Single Server Queue

This is the easiest queuing system as denoted in the below mentioned figure. The central entity of the system is a server, which caters service to the associated Configurable Items (CI). The CIs requests the system to be assisted, in case the server is in idle state. It is then attended instantly, otherwise it gets associated with a waiting queue. After the specific task is accomplished by the server, the entity leaves.

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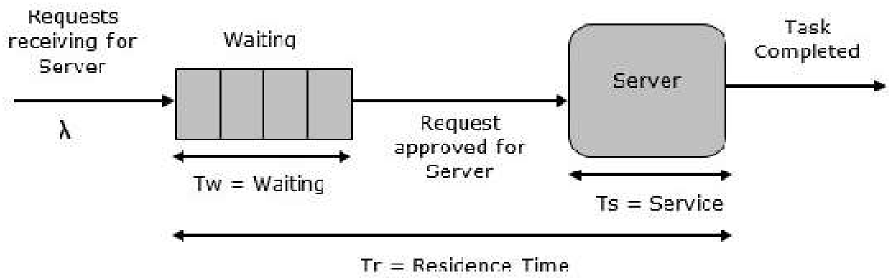

Fig. 2.20 Single Server queue

Multi Server Queue

As per the name, the system comprises of numerous servers and a single shared queue for all entities. When any entity requests service for the specific server, it is assigned to in case at-least one server is available. Else the queue commences to initiate till the server is able to serve. In this case, we take into consideration that all the given servers are same in terms of configuration, i.e., any server can address to serve any request in a similar fashion.

There is an exclusion of consumption. Let **N** be the alike servers, then ρ is the consumption of each server. Consider **Nρ** to be the consumption of the whole system. In such case, the maximum consumption is **N*100%** and maximum input rate is as follows:

$/max = \frac{\text{N}}{\text{T}s}$

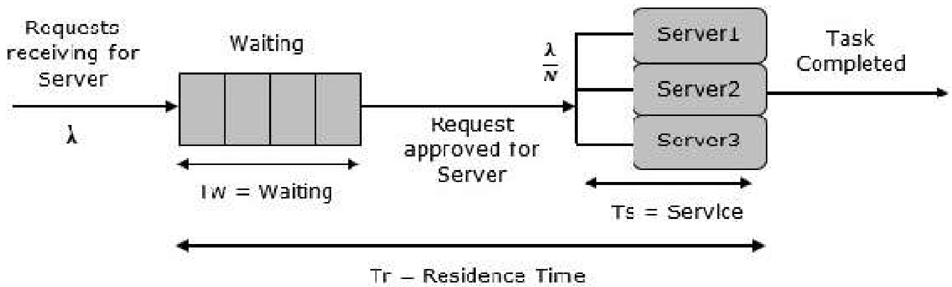

Fig. 2.21 Multi Server Queue

Queuing Relationships

The below table illustrates some rudimentary relationships in the area of queuing.

General Terms	Single Server	Multi server
r = λTr Little's formula	ρ = λTs	ρ = λTs/N
w = λTw Little's formula	r = w + ρ	u = λTs = ρN
Tr = Tw + Ts		r = w + Nρ

NOTES

Simulation of Time-Sharing System

Time-sharing system is fabricated in such a fashion that every user uses a minor share of time shared on a specific system. It helps multiple users to share the single system, concurrently. The swapping of each user is so fast that each user recognizes like using their individual system. It is on the basis of the notion of CPU scheduling and multi-programming where multiple CIs can be taken int consideration efficiently and successfully by executing numerous processes or jobs concurrently on a system.

Example – SimOS Simulation System.

Discrete-Time Models with Difference Equations

Discrete-time models are simple to understand. At the same time, it is easy to develop and perform required simulation. They are effortlessly deployable for stepwise computer simulations, and they are frequently appropriate for modelling experimental data that are nearly continuously now discrete. Moreover, they are able to signify unforeseen fluctuations in the state of the system, and perhaps disordered dynamics, by means of scarcer variables than their continuous-time counterparts. The discrete-time models of dynamical systems are frequently known as Difference Equations, because any ûrst-order discrete-time dynamical system with a state variable can be written easily as xx (Equation 2.1)), i.e.,

$$xt=F(xt-1,t) \quad (2.1)$$

on differentiating

$$\Delta x=xt-xt-1=xt=F(xt-1, t)-xt-1 \quad (2.2)$$

which is mathematically more similar to differential equations.

Note that Equation 2.1can also be written as

$$xt+1=F(xt,t) \quad (2.3)$$

This is equivalent to 2.1 we will use the notation with

xt, xt–1, xt–2xt, xt–1, xt–2, etc., as this notation is easier to see how several preceding steps are desirable to compute the next step.

From a difference equation, a series of values of the state variable x over time can be generated, initiating with preliminary condition x0:

x0,x1,x2,x3....

This is called **time series**. In this case, it is a forecast made using the difference equation model, but in further circumstances, time series likewise denotes consecutive values attained by experiential reflection of real-world systems as well.

Let us put a straightforward example of a discrete-time, discrete-state dynamical system. The system consists of two interacting components: A and B. Each component attains one of two probable conditions, i.e., either Red or Green. Their behaviours are concluded by the following rules:

- A try to remain the same colour as B.
- B tries to be the reverse colour of A.

These rules are certainly applicable to their states concurrently in discrete time steps.

2.7 CONTINUOUS SYSTEM MODELS

NOTES

A continuous system is type of a system where all the involved significant activities of the system get accomplished without further delay. To be more precise, no queue formation takes place for events. Additionally, no sorting of time simulation taken place. When a continuous system is modelled with the help of mathematics, its corresponding variables which represents the associated attributes are completely taken care by continuous functions.

What is Continuous Simulation?

Continuous simulation is a kind of simulation where state variables get changed on a continuous manner with respect to time. Following (Fig. 2.22) is the graphic illustration of its associated behaviour.

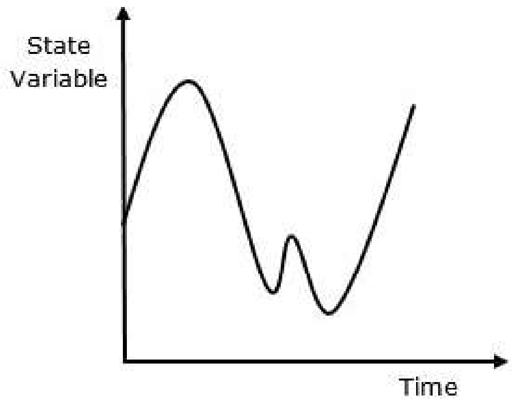

Fig. 2.22 *Continuous Simulation*

Why Use Continuous Simulation?

We need to utilize continuous simulation because it is highly dependent on differential equation of numerous parameters related to the system and its associated projected consequences identified to us.

Application Areas

Continuous simulation is normally used in the following sectors.

- In the area of civil engineering for the building of dam ridge and construction of tunnels.
- In the area of military applications for simulating the trajectory of missile, etc.
- In the area of logistics for fabricating of toll plaza.
- It is used to perform the movement analysis of passengers at the railway station or airport terminal.
- It is used to evaluate flight scheduling.
- It is used to simulate business development for new product. planning of development for new products, management of employees, analysing markets, etc cala so be done via this simulation model.

NOTES

Monte Carlo Simulation

Monte Carlo simulation is a computerized mathematical method to produce random sample data which is on the basis of some recognized distribution for numerical experimentation. This method is normally applicable to risk quantitative analysis and decision-making problems. This method is normally used by people in the area of oil & gas, research & development finance, engineering, project management, energy, manufacturing, insurance, transportation, etc.

This method was first used by scientists working on the atom bomb in 1940. This method can be used in those situations where we need to make an estimate and uncertain decisions such as weather forecast predictions.

Monte Carlo simulation can be used to address an array of challenges in effectively all arena e.g., engineering, finance, supply chain, and scientific research. It is also known as a "multiple probability simulation".

Take away from Monte Carlo Simulation

- Monte Carlo simulation is a model which is referred to during the prediction of the probability of various consequences when the interference of random variables is in existence.
- Monte Carlo simulations provides assistance in elucidating the influence of negative possibilities and uncertainty in the forecasting and foretelling models.
- Various arena uses Monte Carlo simulations for accomplishing a successful simulation.
- The primary arena of Monte Carlo simulation includes allocating numerous values to an uncertain variable to accomplish various outcomes and then performing an average of the outcomes to acquire an approximation or estimation.
- Monte Carlo simulations undertakes seamlessly effectual markets.

Using Monte Carlo simulations, for defining a probable trajectory of price, data related to the past price of the items are normally used for generating an array of Periodic Daily Return via the natural logarithm (this equation will vary from the formula related to the normal percentage change). The formula used to calculate Periodic Daily Return is a follow:

$$ln\left(\frac{\text{Day's Price}}{\text{Previous Day's Price}}\right)$$

Subsequently, functions like AVERAGE, STDEV.P and VAR.P over the complete resultant sequence are used to acquire the average daily return, standard deviation, and variance inputs, respectively. The drift related formula is as follows:

$$\text{Drift} = \text{Average Daily Return} - \frac{\text{Variance}}{2}$$

where:

Average Daily Return = Yield from the calculation

AVERAGE is the function from intermittent daily returns array series

Variance = Produced from calculation

VAR.P is the function from intermittent daily returns array series

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As an alternate approach, the value of drift can also be fixed as 0; this selection echoes a specific theoretical alignment, but it is assured that the variance will never be immense, specifically for the lesser time frames.

The formula to attain a random input is as follows:

Random Value = $\sigma \times$ NORMSINV (RAND)

where:

σ denotes the Standard deviation, produced from calculation

STDEV.P is the function from the periodic daily returns array of series

NORMSINV and RAND are the functions (used in Excel also)

The equation to calculate the next day's price is:

Next Day's Price=Today's Price$\times e^{(\text{Drift+Random Value})}$

Monte Carlo Simulation – Important Characteristics

Following are the three distinctive features of Monte-Carlo method:

- The output of the simulation should generate random samples.
- The distribution of its input should be familiar.
- The resultants should be known during performing the associated experiment.

Monte Carlo Simulation – Advantages

- It is easy to deploy.
- This method delivers statistical sampling for mathematical experiments by means of computer.
- This method offers approximate solution to mathematical problems.
- This method can certainly be used for both stochastic and deterministic problems.

Monte Carlo Simulation – Disadvantages

- This method consumes a huge amount of time as there is a requirement to generate huge number of samples to obtain the opted output.
- The resultants of this method are solely the approximation of the true values and is not the exact one.

Monte Carlo Simulation Method – Flow chart

The following (Fig. 2.23) portrays a widespread flowchart of Monte Carlo simulation.

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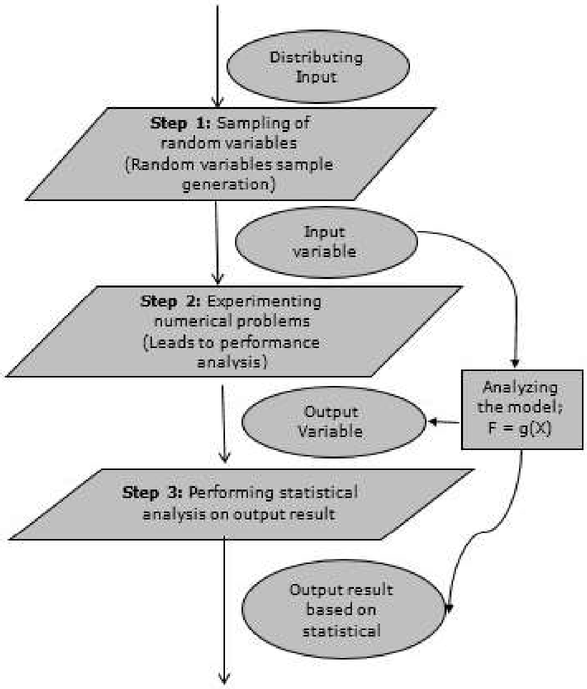

Fig. 2.23 *Portrays a Widespread Flowchart of Monte Carlo Simulation*

Monte Carlo uses the "Law of Large Numbers (LLN)" to bring up with an approximation for a specific populace parameter via simulated values.

Definition of LLN: Let us consider X1, X2, . . . , Xn are all autonomous random variables with the identical primary distribution, which is also known as "Independent Identically-Distributed (IID)", where all values of X consists of identical mean ì and standard deviation σ.

With the growing of the sample size, the probability the average of all values of X is similar to the mean ì and is equal to 1. So, LLN can be defined as:

$$\mathbf{E}\,(X_i) = \mu \quad \mathbf{Var}\,(X_i) = \sigma^2 \quad \text{for } i = 1,\ldots,N$$

$$\overline{X}_n = \frac{\sum_{i=1}^{N} X_i}{N}$$

$$N \to \infty \quad \text{then} \quad \Pr(\overline{X}_n = \mu) = 1$$

Case Study for Monte Carlo Simulation

Monte Carlo simulation is a comprehensive style of calculating algorithms that responses over a reiterated random sampling to acquire mathematical outcomes. Its indispensable thought of considering the randomness to resolve difficulties is certainly regulative in attitude. This approach is mostly used when it is tough or unfeasible to practice other methods. Historical data from NASA depicts that they have used Monte Carlo Simulation for over 100 space missions and in all the cases the simulation has given best result for calculating cost, scheduling and mitigating associated risks.

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Monte Carlo methods is also used to model in defending from cyber-attacks, identifying cyber risks, mitigating vulnerabilities and identifying investments in relation to develop cyber security related software solutions. The model is set is a form of a read life system and millions of cyber threat attacks goes to the system as inputs. The system behaviours are then observed very minutely. The outcomes are notes. These outcomes as interpreted as those areas of the system where vulnerabilities still exist. Based on the vulnerabilities, associated risks are identified and the mitigation solution is embedded in the system to make the system full proof. This kind of tests are repetitive in nature where inputs are required to get changed based on certain circumstances. With the change of the inputs, the outputs also get changed. Averaging the changed output against the inputs the mitigation solution is designed.

With the successes of the outcomes of the above-mentioned simulation, cyber risk which is considered to be a decisive business risk related treatments can be planned and mitigated. Risk treatment includes avoidance of risk, retention or acceptance of risk, sharing of risk, transferring of risk, and reduction of risk. Strategic risk management can be planned accordingly. Along with this kind of simulation risk related to statutory and regulatory requirements can also be mitigated at the basic level. The simulation is also used to calculate the subsequent effect in case the risks are not mitigated. The effects can be the risks related to the market. Simulations and models are also used for the measurement of predictable loss due to cyber-attacks or cyber-incidents, as well as Annual Loss Expectancy, Standard Deviation of Loss, and Perceived Composite Risk. Common Vulnerability Scoring System (CVSS) and Analysis of Information Risk (FAIR) can also be framed with the said simulation methodology. If the model is implemented properly, it intensely helps in calculating the Return on Security Investment (ROSI), Net Present Value (NPV) and Internal Rate of Return (IRR) as well. Decisions for strategies related to risk management can be forecasted. For example, to identify the point of diminishing return over cyber risk investment. When controls based on statutory and regulatory requirement (cyber risk accumulation and aggregation, national digital approaches and policies, cyber regulations consisting of General Data Protection Regulation, CIC, COBIT, NIST Cybersecurity Framework, ISO 27001 & ISO 27002, SOC2, NERC-CIP, HIPAA, GDPR and FISMA) for securing the systems needs to be implemented, the system effect can be concluded with the help of Monte Carlo Simulation.

Elements Comprising of a Simulation Model

- Objects of the model i.e., Entities and Resources.
- Organization of entities and resources i.e., Attributes, State and List.
- Operations of the objects i.e., Event, Activity, Delay and clock.

Trace-Driven Simulation

A trace-driven simulation examines a static arrangement of trace records from a specific file as an input. These trace records normally signify references of memory, branch outcomes, or detailed machine instructions, etc.

- Trace signifies the "Time ordered" related data of occurred consequences on a specific system.

- Trace-driven simulation refers to the input to the trace.
- Used in scrutinizing or fine-tuning resource management algorithms i.e., paging mechanism, analysis of cache, scheduling of CPU, prevention of deadlock, dynamic allocation of storage.

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Advantages of Trace-Driven Simulation

- Trustworthiness
- Validations are easier
- Workloads are precise
- Detailed trade-offs are possible
- Workload in detailed fashion can go as a input
- Randomness is much less in this method
- Comparison can be done impartially

2.8 MODELLING AND SIMULATION PLATFORM

Modelling is a method to fabricate a virtual illustration of a real-world system which consists of software and hardware. When the software constituents of this model are propelled by mathematical relationships, it is possible to simulate this virtual illustration under a extensive series of conditions to observe how it performs.

Modelling and simulation are particularly appreciated for examining various situations that might be tough to replicate with hardware prototypes unaided, primarily in the initial stage of the design progression when hardware is scarce. Reiterating between modelling and simulation can certainly progress the excellence of the system design initially, thus bringing down the number of faults identified in the later stage in the design process.

Usual illustrations for system models consist of block diagrams, schematics, and state diagrams. With the help of these representations, model for mechatronic systems can be established. Additionally, communications systems, control software, algorithms, and signal processing can also be modelled.

2.8.1 SIMSCRIPT

Without computers, it is impossible to accomplish any accurate and dynamic systems simulation.

- SIMSCRIPT was developed in 1962 by Nobel Laureate Harry Markowitz.
- It was developed to simulate an Air Force RAND project.
- It is a free-form
- It is powerful and English like
- It permits long identifiers
- Users can tailor the source code appearance
- Consists of programming language and auxiliary structures

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- It supports the object-oriented simulation development.
- It consists of Interactive Development Environment (IDE) SimStudio.
- Supports Windows and UNIX environment. Programmable on 32-bit and 64-bit Linux environment.
- Supports database interfaces
- Support over 33 years
- Programs are easy to read and modify
- Generate result which are adequate to users to interpret.
- It does not require any coding in any other languages. It itself is sufficient enough.
- Model components can be coded very easily
- It consists of built-in facilities for the development of a model
- Has built-in debugger to debug codes which are automatically detected as faulty.

Program Structure of SIMSCRIPT

Entities

Permanent

Temporary

Sets

Event routines

SIMSCRIPT Control Structures

- Assignments
- Branches
- Loops
- Input
- Output
- Routines and functions

SIMSCRIPT Program Structure

The SIMSCRIPT program contains of three main elements. They are:

- A **preamble** providing the static explanation of every modelling component.

It is absolutely declaration type in nature. No executable statements are included here.

Each and every modelling component e.g., processes and resources are declared in this preamble.

- A **main program** where the associated execution starts.

Resources should be created and initialized prior to they can be used by the processes.

This is typically proficient in the MAIN section.

- A **process/event routine** for every process/event announcement stated in the preamble.
- **Event notices**: These are formed when it is decided that an event should be scheduled.

Every event notice jeep tracks the time, the event is owing to spawn and the event routine that is to spawn the event.

- **Names**: Names normally contain any amalgamation of alphabets and digits provided in the program. There should be at least one alphabet. Period (".") can be used.

Example: NAME, XY12, XYZ.ABC

- **Labels:** Labels are normally used for recognizing the programming statements. It contains any amalgamation of alphabets and numbers. Period (".") can also be used.

  Labels are used between single quotation marks.

  Example: 'LABEL1', 'XY12', '534132'

  SIMSCRIPT Statements

  # Statements for printing:

  PRINT n LINES AS FOLLOWS

  PRINT n LINES THUS

  # Printing with values of a specific variable:

  PRINT n LINES WITH X AND Y LINE THIS

3 nos numeric integer * * *, 4 nos numeric real number with one decimal place should be designated by * * * .*

A Sample SIMSCRIPT Program with Arithmetic Expressions

Arithmetic expressions are designed by combination of variables and constants with arithmetic operators.

The arithmetic operators are + (add), - (subtract), * (multiply), / (divide), and ** (exponentiate).

Example:

let R = L+M/(P-X*Y)**2

is calculated in the following way:

1. Multiply X by Y.
2. Subtract the product from P.
3. Square the difference.
4. Divide M by the result of 3.
5. Add L.
6. Store the result in R.

A Simple Example of a SIMSCRIPT Program: Read X and Y

NOTES

add A to B

print 1 line with Y thus

The Sum is: *****

stop

This program contains of four statements.

1. Read the values of two variables called A and B from input data
2. Sum these variables
3. Print the sum of the variables along with the message, "The Sum is:"
4. Stop.

In the example, read, and, add, to, print, line, with, thus, and stop are all SIMSCRIPT keywords and should never be changed.

A, B, and the variables

The Sum is: are called expressions which is given by the developer.

Variables in SIMSCRIPT

- **Variables**

Integer	Let NUMBER = 10
Real, Double	Let AMOUNT = 7.25
Alpha	Let CLASS = "X"
Text	Let ROUTE = "Sample text or Sentences"

- **Reading Input Data**

  The general form of a read statement is:

  read variable name list

Example:- read X, Y and QUANTITY

Printing Statements

The variables AMOUNT and ITEMS appearing in the following print statements are assumed to have the values and 27, respectively:

1. print 1 line with AMOUNT/ITEMS thus

AMOUNT/ITEM = Rs.*.***

is printed as:

PRICE/ITEM = Rs. 8.036

2. print 3 lines with AMOUNT, ITEMS, AMOUNT/ITEMS thus

AMOUNT = Rs.***.*

ITEMS = *

AMOUNT = Rs.200.9

ITEMS = 25

AMOUNT/ITEM=Rs.8.036

If Statement

if statement is used to assess the truthiness or falseness of a logical expression.

The syntax of the if statement is as follows:

if logical expression

first series of statements

else

second series of statements

Always

Example:

if GRADE = MASTER

add 5 to QUEUE

let GRADE = MASTER

always

The for loop

For loop

for I = 1 to 5 by 1

print 1 line with I and I ** 2 thus

THE SQUARE OF * IS *

output

THE SQUARE OF 1 IS 1

THE SQUARE OF 2 IS 4

THE SQUARE OF 3 IS 9

THE SQUARE OF 4 IS 16

THE SQUARE OF 5 IS 25

Nested Loop

Nested Loop for NUMB = 1 to 12, for MULTI = 1 to 10,

print 1 line with MULTI, NUMB, and MULTI * NUMB thus

** TIMES ** IS ***

o/p

1 TIMES 1 IS 1

2 TIMES 1 IS 2

3 TIMES 1 IS 3

10 TIMES 1 IS 10

1 TIMES 2 IS 2

2 TIMES 2 IS 4

3 TIMES 2 IS 6

NOTES

NOTES

Array

for X = 1 to N

read LIST(X)

A comment is normally delimited on the left side by two single apostrophes (‘’).

Sample Program Using SIMSCRIPT

```
preamble
      usually, mode is integer
      usually, mode is real
end
main
      read X
      read N1 and N2
      call PROCESS.DATA
routine PRINTOUT
      print 2 line thus
      Hello He is in India
return
```

SIMSCRIPT Time Routine

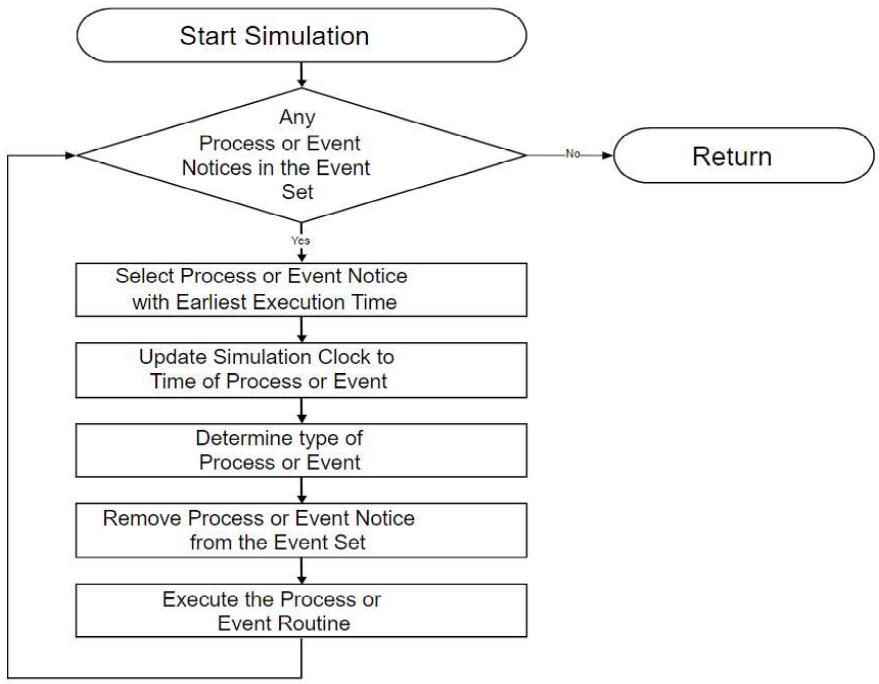


***Fig. 2.24** Flowchart of SIMSCRIPT Time Routine*

SIMSCRIPT Representation of Time

```
preamble
define .SECONDS to mean DAYS
define .MILLISECONDS to mean HOURS
define .MICROSECONDS to mean MINUTES
end
main
let HOURS.V = 1000
let MINUTES.V = 1000
end
```

2.8.2 GPSS

Following are the features of GPSS:

- GPSS stands for General Purpose Simulation System.
- IBM's Geoffrey Gordon developed GPSS during 1960s.
- GPSS is a distinct time simulation language where distinct stages are progressed via a simulation clock.
- A system is modelled as transactions (processes) i.e., it makes an entry into the system and are passed from one service to another.
- GPSS is a software package that is designed to shorten complex situations for user.
- User translates their problem into a conceptual model, inform of a block diagram.
- Then GPSS software:
  - o Processes this block diagram
  - o Executes the simulation run
  - o Generates the statistical outputs
- It is generally used for modelling traffic and queuing systems.
- It is not necessary to write programs in GPSS like SIMSCRIPT
- GPSS works on the basis of blocks. Block is the block diagram which is the network of interconnected blocks.
- GPSS provides a set of 48 various blocks. Each block has a name and possess an activity
- GPSS can be applicable efficiently and effortlessly to a comprehensive range of problems.
- Predominantly suitable for problems, e.g., a optimization a factory operation
- There are various versions of GPSS. They are:
- GPSS/360, GPSS II, GPSS III, Flow Simulator, GPSS K, GPSS V, GPSS/PC, GPSS World.

NOTES

- GPSS is an event simulator which primarily looks into system dynamics via queues, storages, etc.

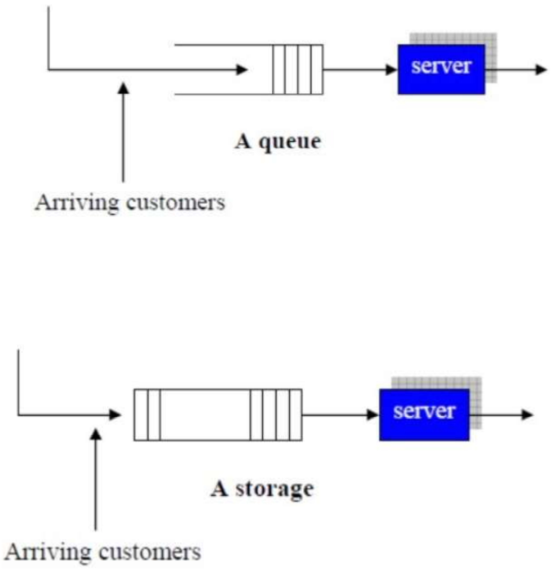


Fig. 2.25 *General Purpose Simulation System*

Basic Structure of GPSS

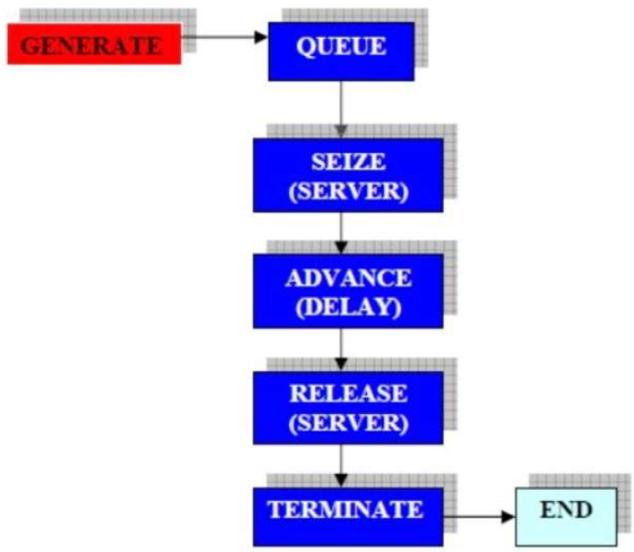


Fig. 2.26 *Basic Structure of GPSS*

Block

- Block is the fundamental structural element of GPSS.
- A GPSS model is provided by its corresponding block diagram.
- Example of some blocks are as follows:
  - GENERATE
  - TERMINATE
  - ASSIGN

- SEIZE
- RELEASE
- QUEUE
- DEPART
- ADVANCE
- START
- END

Transaction

- Transaction is a process which characterizes the real-world system which is being modelled.
- Transactions are done by moving from block to block.
- Each transaction in the model consists of one block
- One single block might consist of multiple transactions.

Operands of Blocks

- Some GPSS blocks with their operands are as follows:
  – GENERATE L, M, N, O, P
  – SIEZE L
  – QUEUE L

Generating and Terminating Transactions

- GENERATE block generates transactions
- TERMINATE block destroys the transactions

Using Facilities

- GPSS provides the facility modelling concept to characterize restricted readiness of a service.
- A **facility** is a type of resource which can only be used at a single given point of time by the transaction.
- To demand for a facility, the associated or corresponding transaction should enter in SEIZE block.
- Once a transaction is entered into the SEIZE block, it possesses the facility and other transactions can't enter the specific block.
- This is known as **blocking**.
- When a transaction no longer requires the facility, it has taken,
  - o The transaction immediately releases the ownership of the facility, making it available to be seized by some other transactions.
- The RELEASE block is used for the realising the facility.

Blocking Sample

Only one transaction at a time is allowed to own the facility MECHANIC

GENERATE 300, 20

SEIZE MECHANIC

ADVANCE 280,200

RELEASE MECHANIC

NOTES

TERMINATE 1

SEIZE A

• Waits for or acquires ownership of a Facility Entity.

A: Facility name or number.

SEIZE Teller1

SEIZE Server1

RELEASE A

• Releases ownership of a facility.

A: Facility name or number.

RELEASE Teller1

RELEASE Server1

Case Study - Barber Shop Simulation

Barber shop simulation GPSS block diagram from General Purpose Simulation System

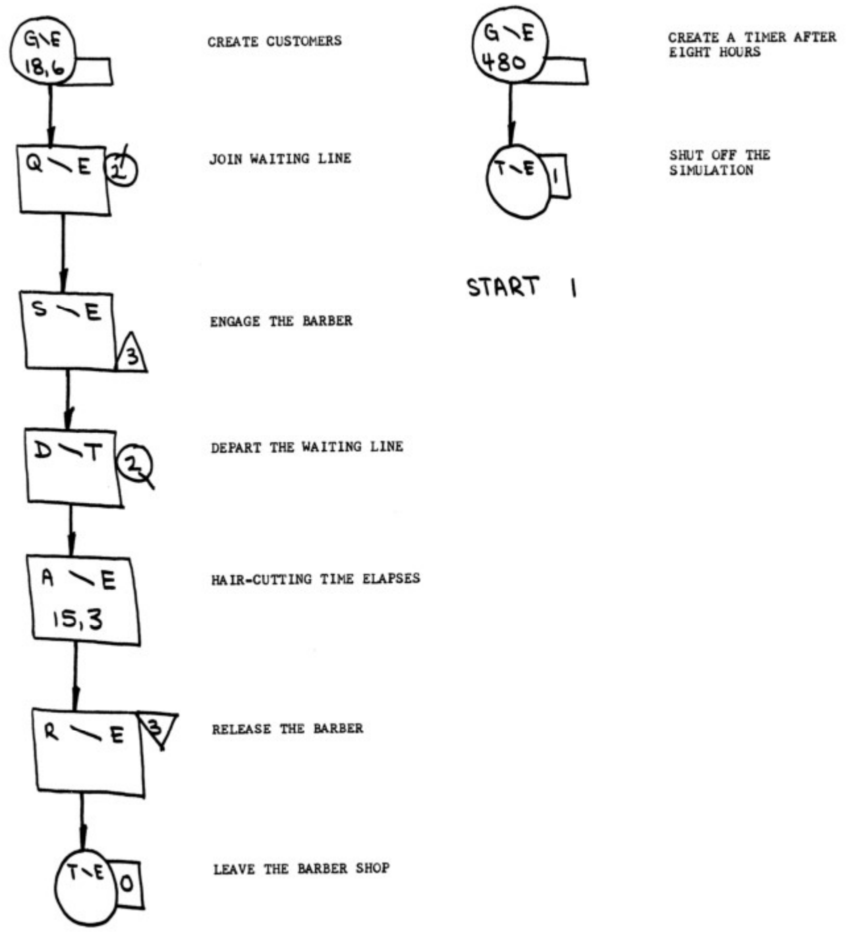


Fig. 2.27 First Model from General Purpose Simulation System

NOTES

We are modelling a barber shop with the following qualities:

- The shop contains one barber and one barber's chair, open for eight hours in a day.
- Customers arrive on average every 18 minutes, with the arrival time varying between 12 and 24 minutes.
- If the barber is busy, the customer will wait in a queue.
- Once the barber is free, the next customer will have a haircut.
- Each haircut takes between 12 and 18 minutes, with the average being 15 minutes.
- Once the haircut is done, the customer will leave the shop.

We want to answer these questions:

- How utilised is the barber through the day?
- How long does the queue get?
- On average, how long does a customer have to wait.

The manual uses a block diagram notation to show how this is simulated - you can see this at the top of this page. The equivalent GPSS program looks like this:

```
SIMULATE
    GENERATE    18,6  CREATE CUSTOMERS
    QUEUE       2     CUSTOMERS QUEUE UP IF NECESSARY
    SEIZE       3     ENGAGE THE BARBER WHEN HE BECOMES
                      AVAILABLE
    DEPART      2     CUSTOMER LEAVES THE QUEUE
    ADVANCE     15,3  CUSTOMER GETS HIS HAIR CUT
    RELEASE     3     RELEASE THE BARBER
    TERMINATE   0     LEAVE THE BARBER SHOP
    GENERATE    480   GENERATE A TIMER AFTER 8 HOURS OF
                      SIMULATED TIME
    *
    TERMINATE   1     SHUT OFF THE RUN
    START       1     CARRY OUT THE SIMULATION
    END               RETURN CONTROL TO THE OPERATING SYSTEM
```

The text at the end of each line is a comment; starting a line with * means the rest of the line is a comment; we use this just to separate the two parts of the program as explained below.

GPSS works in units rather than absolute time, so we shall say that 1 time unit is equal to 1 minute. Programs can be read from top to bottom: the simulator runs by advancing the clock one time unit and working out where transactions are in the system.

The first line, SIMULATE, denotes the start of the code.

GENERATE 18,6 means generate a transaction - a barber shop customer - every 18 minutes ± 6 minutes.

NOTES

QUEUE 2 defines a queue with ID 2, denoting the queue where customers will wait.

SEIZE 3 defines a facility with ID 3. The facility is the barber and this line means if the barber is free, the next customer occupies the barber until released.

DEPART 2 says that the customer leaves the queue when occupying the barber.

ADVANCE 15,3 means that transactions in this state only move on after 15 minutes ± 3 minutes – modelling the time taken for a haircut. After that RELEASE 3 shows that the customer no longer occupies the barber and TERMINATE 0 ends the transaction, showing that the customer has left the shop.

That is all that is needed for the basic simulation, but if run like this it would never stop, as we have not modelled the 8 hour period the shop is open.

To do this we generate a new transaction with GENERATE 480, which means generate a transaction after 480 minutes, i.e. 8 hours. The next line, TERMINATE 1, stops the simulation after this transaction is generated.

With that done, the last two lines start the simulator and returns control to MTS when it is done.

Running the program using *GPSS

*GPSS takes the input file on scards and outputs to *sink* by default. Assuming the simulation defined above is stored in barb1.gpss, this is how it would look:

We start off with the program listing, where GPSS denotes which lines are components of the simulation with a block ID. So GENERATE 18,6 is on line 3 but is block 1.

```
$run *gpss scards=barb1.gpss

Execution begins   16:07:50

    GPSS/360   VERSION 1  MODIFICATION LEVEL 4   MTS MODEL
(AN104)   16:07:50  11-03-17

     LINE# CARD#  BLOCK#  *LOC   OPERATION  A,B,C,D,E,F,G
COMMENTS

     1.000  1               SIMULATE

   2.000  2     * BARBER SHOP SIMULATOR

   3.000  3  1  GENERATE   18,6                   CREATE
CUSTOMERS

   4.000  4  2  QUEUE       2                   CUSTOMERS
QUEUE UP IF NECESSARY

   5.000  5  3  SEIZE       3                   ENGAGE THE
BARBER WHEN HE BECOMES AVAILABLE

   6.000  6  4  DEPART      2                   CUSTOMER
LEAVES THE QUEUE

   7.000  7  5  ADVANCE    15,3              CUSTOMER GETS
HIS HAIR CUT

   8.000  8  6  RELEASE    3                    RELEASE
THE BARBER

   9.000  9  7  TERMINATE  0                    LEAVE  THE
```

```
BARBER SHOP
    10.000    10           *
    11.000    11  8        GENERATE   480       GENERATE  A
TIMER AFTER 8 HOURS OF SIMULATED
    12.000    12  9        TERMINATE  1          SHUT  OFF
THE RUN
    13.000    13                              START     1
   CARRY OUT THE SIMULATION
    14.000    14                              END
RETURN CONTROL TO THE OPERATING SYSTEM
```

NOTES

Next, GPSS shows how many transactions were in each block at the end of the simulation and in total.

```
RELATIVE CLOCK         480  ABSOLUTE CLOCK         480
 BLOCK COUNTS
 BLOCK CURRENT   TOTAL   BLOCK CURRENT   TOTAL   BLOCK
CURRENT   TOTAL   BLOCK CURRENT   TOTAL   BLOCK CURRENT
TOTAL
     1       0        27
     2       0        27
     3       0        27
     4       0        27
     5       1        27
     6       0        26
     7       0        26
     8       0         1
     9       0         1
```

We can see that 27 transactions were generated in block 1, which means 27 customers entered the shop during the day. Blocks 8 and 9 had 1 transaction, which is expected as this was the timing transaction. Why does block 5 have one current transaction while all others are zero? Block 5 is the ADVANCE statement, representing the haircut, and this means one customer was in the chair at the end of the simulated day.

GPSS then prints out statistics on the facility, or the barber in our simulation:

```
 FACILITY         AVERAGE          NUMBER         AVERAGE
SEIZING    PREEMPTING
                UTILIZATION      ENTRIES       TIME/TRAN
TRANS. NO.  TRANS. NO.
     3        .816            27        14.518          1
```

So our barber was used 81.6% of the day, and each haircut took on average 14.518 minutes.

We then see details of the queue:

```
 QUEUE       MAXIMUM      AVERAGE      TOTAL        ZERO
PERCENT      AVERAGE      $AVERAGE      TABLE      CURRENT
             CONTENTS     CONTENTS    ENTRIES      ENTRIES
ZEROS      TIME/TRANS    TIME/TRANS    NUMBER     CONTENTS
     2          1       .085       27        16       59.2
1.518         3.727
```

```
 $AVERAGE TIME/TRANS = AVERAGE TIME/TRANS EXCLUDING ZERO
ENTRIES
```

NOTES

MAXIMUM CONTENTS means there was never more than one customer in the queue. Although 27 customers entered the queue, 16 were zero entries - this means they entered and immediately left the queue, i.e., the barber was unoccupied when they entered the shop. For those that did wait in the queue, the average wait was 3.727 minutes; including the zero entries, the average time for all customers to wait was 1.518 minutes.

Finally, GPSS prints usage statistics and exits:

```
CPU TIME USED (SECONDS)
  ASSEMBLY:      .001
TOTAL BLOCK EXECUTIONS:     189
MSEC/BLOCK AVG CPU TIME:   .003
Execution terminated  16:07:50  T=0.004  RC=0  $0.00
```

It's interesting to compare this to the printout in the manual from 1968. Our run took 0.004 seconds of CPU time, compared to 4.5 seconds then.

2.8.3 CSMP III

- CSMP III stands for Continuous System Modelling Program IlI.
- It is an primary scientific computer application which was designed for modelling and resolving differential equations in a numerically way.
- This application was used to empower and live systems to be simulated and tested with the computing power.
- In simple language CSMP uses input in the form of statement type for digital computing and permits a problem to program unswervingly or straight away from a mathematical model in terms of an equation.
- The CSMP was developed in the early 1960s by IBM. This application was hardware dependent. To make it work with another platform as well IBM developed CSMP/FON. CSMP/FON is an open-source solution and can be downloaded from the internet for further use.
- The CSMP/FON offers an integrated and user-friendly atmosphere for creating, testing, and analysing the continuous system models.
- The CSMP/FON simulation application permits users to develop comprehensive interactive simulations in three subsequent steps:
  - By depicting the mathematical model which is to be used in the program.
  - By developing the respective user interface by means of the out of the box graphical elements.
  - By associating the specific properties of those elements to the various variables of the specific model.

Example of CSMP/FON

Each existing element stipulates the relation of three input variables e_1, e_2, e_3 and three parameters p_1, p_2, p_3. The output is a scalar unit whose value is dependent on the tangible relation f for that specific element as follows:

NOTES

$$e_0=f(e_1, e_2, e_3, p_1, p_2, p_3).$$

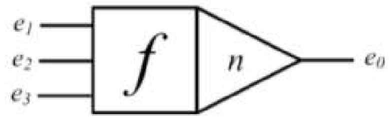


Fig. 2.28 *Portrays the Graphic Display of Elements in a Generic Form*

The Operating principle of the CSMP/FON is illustrated below (Fig. 2.29).

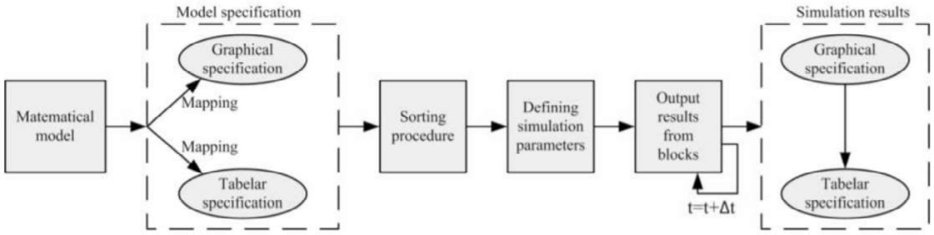


Fig. 2.29 Operating Principle of the CSMP/FON

CSMP/FON Architecture

The CSMP/FON Architecture is provided in the following (Fig. 2.30):

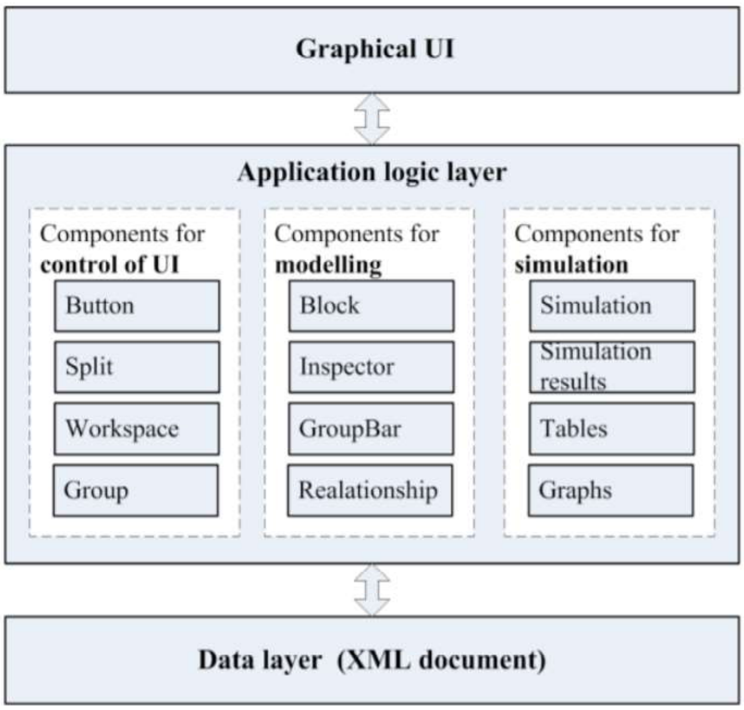


Fig. 2.30 *CSMP/FON Architecture*

Data Layers in CSMP/FON

Models developed using the CSMP/FON are saved as XML files. The structure of this XML file (with the extension. CSMP) is portrayed in Figure 2.31.

NOTES

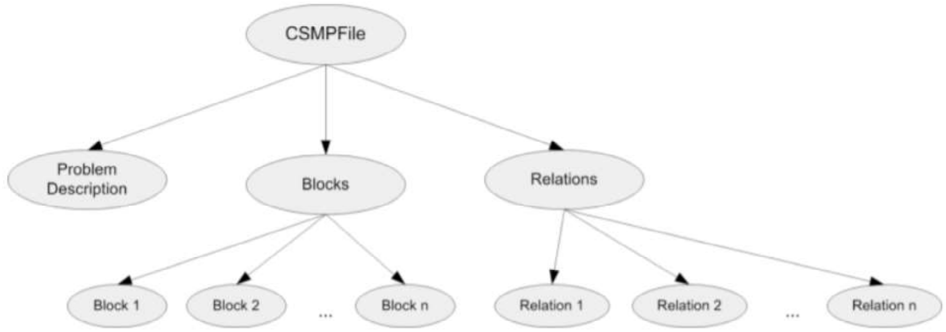


Fig. 2.31 *Data Layers in CSMP/FON*

Sample CSMP/FON Block Diagram

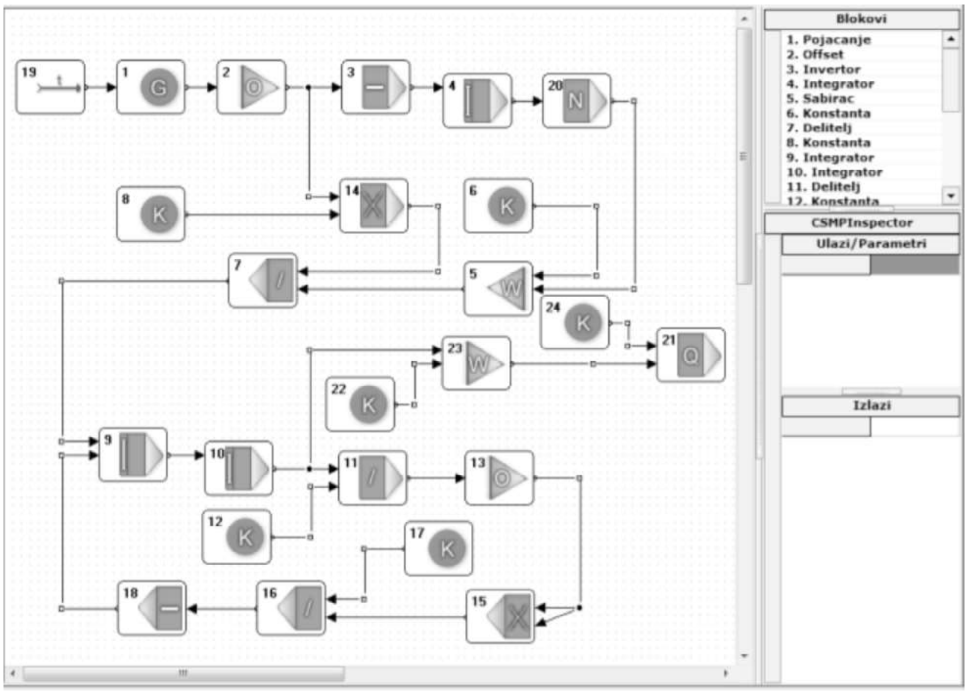


Fig. 2.32 *Sample CSMP/FON Block Diagram*

Statements in CSMP III

It comprises of three types of statement.

I. Structural Statement

II. Data Statement

III. Control Statement

Structural Statement: It describes the model. The structural statement entails statement like FORTAN and associated functional blocks.

Data Statement: These statements are used to allocate numerical values to the associated parameters, initial conditions and various constraints as applicable.

Control Statement: These statements stipulate options in assembly, performs the execution of the written program and the selection of the outputs.

Structural statements is used to perform arithmetical actions e.g. addition, subtraction, multiplication, division and exponentiation, by the means of the identical representation and syntaxes which is similar to the syntaxes used in FORTRAN. For instance, in case, the model comprises of the following equation

Example: X = 6Y/W+ (Z-2) ^2

the following statement shall be used to describe the variables and calculus achieved:

X = (6.0*Y)/W+ (Z-2.0) **2.0

Functional blocks

There are various functional block's.

1. INTEGRATOR

$$Y=\int_0^t Xdt + Ic$$

Functional block:

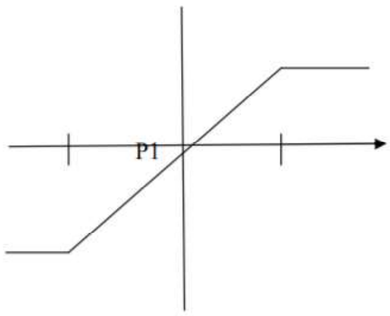
Y=INTGRL(IC, X)

Y (0) =IC

2. LIMITOR: It is used to limit a function within some range.

Y=LIMIT (P1, P2, X)

P2:



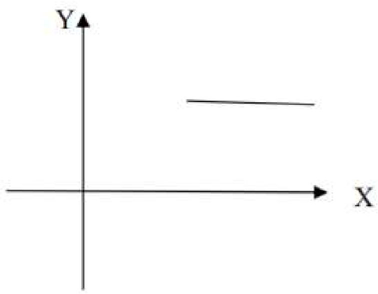
Y=P1, X<P1

Y=P2, X>P2

Y=Y=X1P1 $\leq$ X $\leq$ P2

***Fig. 2.33** Functional Block Diagram*

Step Functions



Y=STEP (P)

Y=SIN(X)

Y=ABS(X)

NOTES

Data Statement

- **Icon**

  It is used to establish the initial values of the function of integration.

  CONST A=0.5

- **PARAM**

  It is used to define a array of values within a specific parameter.

  PARAM D= (0.25, 0.50, 0.75, 1.0)

  CONTROL STATEMENT

- **TIMER**

  To define specific time interval, TIMER is used.

For example: -

TIMER DELT = 0.05 (Integration interval)

FINTIM = 1.5 (Finish time)

PRDEL = 0.1 (Interval at which to print result)

OUTDEL = 0.1 (Interval at which to print-plot)

- In case printed or print-plot output is obligatory, control statement with the keyword PRINT and PRTLT are specified followed by the names of the used variable.
- TITLE, LABLE is used to insert heading on the PRTLT output.

For example: CSMP III program for an automobile suspension system.

Mx + Dx + Kx = KF(t)

Answer: - Mx =KF (t) – Dx –Kx

X =1/M {KF (t) – Dx -Kx}

```
TITLE AUTOMOBILE SUSPENSION SYSTEM

*
PARAM D= (5.656, 16.968, 39.582, 56.56)
*
CONST M = 2.0, F = 1.0, K = 400.0
X2DOT = (1.0/M) * (K * F – D* XDOT – K * X)
XDOT = INTGRL (0.0, X2DOT)
X = INTGRL (0.0, XDOT)
TIMER DELT = 0.005, FINTIM = 1.5, PRDEL = 0.05, OUTDEL = 0.05
PRINT X, XDOT, X2DOT
PRTPLT X
```

```
LABLE DISPLACEMENT VERSUS TIME
END
STOP
```


For example: CSMP-III PROGRAM TO SOLVE THE EQUATION

$3\ddot{X}+15\ddot{x}+50\dot{x}+200x=10$

$\ddot{x}=\ddot{x}=\dot{x}=x=0$ at t=0

*

$3\ddot{X}+15\ddot{x}+50\dot{x}+200x=10$

$3\ddot{X}=10-15\ddot{x}-50\dot{x}-200x$

$\ddot{X}=-1/3(15\ddot{x}+50\dot{x}+200x-10)$

```
*
CONST  M=-1/3,F=1.0,K=-10
X3DOT=(1.0/-3.0)*{(15.0*X2DOT)+(50.0*XDOT)+(200.0*X)-10}
X2DOT = INTGRL (0.0, X3DOT)
XDOT = INTGRL (0.0, X2DOT)
 X = INTGRL (0.0, XDOT)
TIMER DELT = 0.005, FINTIM = 1.5, PRDEL = 0.05, OUTDEL = 0.05
PRINT X, XDOT, X2DOT
PRTPLT X
END
STOP
```

Feature of Software Simulation

- Must support modelling flexibility. Modelling flexibility is the capability to come down to the lower programming levels and supports creation of tailor-made model.
- The application should be user friendly.
- The application supports hierarchical modelling i.e., sub-models consisting of Sub-model.
- Execution speed should be fast enough.
- Should permit to create templates for modelling.
- Should support run-time version for broad delivery of model.
- The application should support importing or exporting of data from/to other similar or dissimilar tools.
- Should be enabled with automatic execution of models for various combination of parameter (input parameters).

NOTES

- Should support combined discrete or continuous modelling.
- Should support 2D and 3D models.
- CAD files can be imported in this application.
- The application should be able to exhibit statistics and graphs, dynamically at the time of execution.
- Should be able to satisfactorily generate random-number for basic U(0, 1) variants.
- Support statistical properties e.g., cycle length, adequate streams and sub-streams.
- The software should allow to create self-governing replications.
- Should support Object Oriented programming and Object-Oriented simulation for OOPS programming.
- Digital thread for product development can be created. This will drive the script alliance, communization of process, traceability of data and dashboards for processing and the product KPIs for simulating through the entire product lifecycle.

Check Your Progress

10. What are the three basic simulation modelling software?
11. How will you define the discrete systems model?
12. What is continuous simulation?
13. In which year was SIMSCRIPT language developed?
14. Write the some features of GPSS.
15. How will you define the CSMP III?

2.9 ANSWERS TO 'CHECK YOUR PROGRESS'

1. System modelling is the procedure of building immaterial, intangible or abstract models of a specific system.
2. The various types of UML diagrams are:
   - Activity diagrams
   - Sequence diagrams
   - Class diagrams
   - State diagrams
3. The various kind of models in UML are:
   - Context models
   - Interaction models
   - Structural models
   - Behavioural models
   - Model-driven engineering

NOTES

4. Use cases was originally developed to provide support for requirements induction and currently it is fused into UML.
5. Structural models portray the establishment of a system in terms of the various components that the system consists of along with their relationships.
6. Model-Driven Engineering (MDE) is a method to software development. The models are preferred to rather than programs. From this model it the developed programs are execute on a specific platform. Model-driven engineering is still at an initial phase of development.
7. The benefits of simulation are as follows:
   - Establishing a risk-free environment
   - Saving time and cash
   - Picturing or Visualization
   - Understanding the dynamics
   - Accuracy to the fullest extent
   - Handle uncertainty efficiently
8. Simulation models comprise of the following components are:
   - System entities
   - Input variables
   - Performance measures
9. Modelling & Simulation are applicable for various areas. They are:
   - Applications related to defence
   - Training and support
   - Designing of semiconductors
   - Applications related to telecommunications
   - Field of civil engineering designs and its corresponding presentations
   - Business models related to E-business
   - Internal structure of a complex biological system
   - Routing algorithms
   - Assembly line
   - Testing of new designs
   - Testing of policies
   - Verification of analytic solutions
10. Simulation modelling software normally considers the following 3 worldviews:
    - Scheduling of events
    - Interaction with processes
    - Scanning of associated activities
11. In discrete systems, the modifications in the system state are intermittent and every change happening within the state of the system is known as an event. The model which is used in a discrete system simulation normally

consists a set of numbers to characterize the situation of the system and is known as the state descriptor.

12. Continuous simulation is a kind of simulation where state variables get changed on a continuous manner with respect to time.

NOTES

13. SIMSCRIPT was developed in 1962 by Nobel Laureate Harry Markowitz. It was developed to simulate an Air Force RAND project.
14. Some GPSS features are:
    - GPSS stands for General Purpose Simulation System.
    - IBM's Geoffrey Gordon developed GPSS during 1960s.
    - GPSS is a distinct time simulation language where distinct stages are progressed via a simulation clock.
15. CSMP III is a primary scientific computer application which was designed for modelling and resolving differential equations in a numerically way.

2.10 SUMMARY

- System modelling is the procedure of building immaterial, intangible or abstract models of a specific system.
- A model can be developed for an external perspective, where the context or environment of the system is exemplified or modelled.
- System modelling is a norm representation of a system which is accomplished by using certain of graphical representation. These graphical notations are based on symbolizations which is used in the Unified Modelling Language (UML).
- Context models are used to demonstrate the operational framework of a specific system – it demonstrates what are laying outside of the system restrictions.
- Use cases was originally developed to provide support for requirements induction and currently it is fused into UML.
- Sequence diagrams is a portion of UML. It is used to determine the various interactions between the actors and the various objects within a system.
- Model-Driven Engineering (MDE) is a method to software development. The models are preferred to rather than programs. From this model it the developed programs are execute on a specific platform. Model-driven engineering is still at an initial phase of development.
- Experimental frame is used to learn a system in the production environment. It consists of various investigational conditions, characteristics, purposes, etc. The Basic Experimental Frame comprises of two sets of variables.
- Lumped Model is a precise description of a system and it maintains the quantified conditions of a specified experimental frame.
- A scale model is a bodily depiction of lesser magnitude, often with only indispensable particulars (e.g., map of a country or a model of a ship).

- A mockup is a full-scale depiction, typically overlooking unseen aspect (e.g., interior of a ship cabin, an aircraft control panel).
- An analog forecast the behaviour of an analogous system.
- Symbolic models are the abstract illustration for the system under the purview. A symbolic model can also be intuitive as well or can also be logical.
- The modifications in the system state are intermittent and every change happening within the state of the system is known as an event. The model which is used in a discrete system simulation normally consists a set of numbers to characterize the situation of the system and is known as the state descriptor.
- Continuous simulation is a kind of simulation where state variables get changed on a continuous manner with respect to time.
- Monte Carlo simulation is a model which is referred to during the prediction of the probability of various consequences when the interference of random variables is in existence.
- GPSS provides the facility modelling concept to characterize restricted readiness of a service.

2.11 KEY TERMS

- **System Modelling:** It is the procedure of building immaterial, intangible or abstract models of a specific system.
- **Sequence Modelling:** It is a portion of UML. It is used to determine the various interactions between the actors and the various objects within a system.
- **Entities & Attributes:** An entity signifies an object. Its value can be static or dynamic in nature. This value is dependent on the process with other entities. Attributes are always local values which are utilized by the entity.
- **Subsystem Validity:** It is not necessary that a model should have any existing system to compare it with, but there may be a situation where there might exist a known subsystem.
- **Time Slicing:** It is the time which is demarcated by a model for apiece event till the nonappearance of any event.
- **Continuous Simulation:** It is a kind of simulation where state variables get changed on a continuous manner with respect to time.
- **SIMSCRIPT:** It was developed in 1962 by Nobel Laureate Harry Markowitz. It was developed to simulate an Air Force RAND project.

2.12 QUESTIONS AND EXERCISE

Short-Answer Questions

1. Define the term system modelling.
2. What is UML diagram?

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3. State about the sequence diagrams.
4. How will you define the simulation?
5. What do you mean by the experimental frame?
6. What are the key features of discrete event simulation?
7. Define multi server queue.
8. Learn about the continuous simulation.
9. What is SIMSCRIPT?
10. Define the term GPSS.
11. How will you define the CSMP III?

Long-Answer Questions

1. Discuss briefly about the system modelling with the help of examples.
2. Explain briefly about the UML. Give appropriate examples.
3. Differentiate between the data-driven modelling and event-driven modelling with the help of relevant examples.
4. What do you understand by the simulation with the help of examples? Explain.
5. Discuss the advantages and disadvantages of modelling & simulation. Give appropriate examples.
6. Briefly explain about the discrete system models with the help of examples.
7. Discuss briefly about the parameters used in the queuing system. Give appropriate examples.
8. Explain briefly about the continuous system models with the help of examples.
9. Illustrate briefly about the SIMSCRIPT. Give appropriate examples.
10. Discuss about the GPSS with the help of relevant examples.
11. Explain briefly about the CSMP III with the help of examples.

2.13 FURTHER READING

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UNIT 3 MODEL VERIFICATION AND VALIDATION

NOTES

Structure

3.0 INTRODUCTION

Verification and validation (also abbreviated as V&V) are independent procedures that are used together for checking that a product, service, or system meets requirements and specifications and that it fulfills its intended purpose. These are critical components of a quality management system such as ISO 9000. The words "Verification" and "Validation" are sometimes preceded with 'Independent', indicating that the verification and validation is to be performed by a disinterested third party. "Independent verification and validation" can be abbreviated as "IV&V". Verification is the procedure of examination and scrutinizing that a software accomplishes its target without any errors. It is the procedure to guarantee whether the application which is developed is appropriate or not. It validates whether the developed application achieves the necessitates that has been defined as requirement. Verification is always a static testing and validation is the procedure of inspecting whether the developed application is up to the mark or not. It is the procedure of scrutinizing the validations of the developed application. In other words, it inspects whether the validations are able to produce the correct output as per expectations Validation is always a dynamic testing.

In this unit, you will learn about the validation and verification, estimation methods, simulation run statistics, replication of runs and regenerative techniques.

3.1 OBJECTIVES

After going through this unit you will be able to:

- Understand the basic of validation and verification model.
- Analyse the estimation methods
- Discuss about the simulation run statistics

- Define replication of runs
- Learn about the regenerative techniques

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3.2 VALIDATION AND VERIFICATION

Verification is the procedure of examination and scrutinizing that a software accomplishes its target without any errors. It is the procedure to guarantee whether the application which is developed is appropriate or not. It validates whether the developed application achieves the necessitates that has been defined as requirement. Verification is always a static testing.

Validation is the procedure of inspecting whether the developed application is up to the mark or not. It is the procedure of scrutinizing the validations of the developed application. In other words, it inspects whether the validations are able to produce the correct output as per expectations Validation is always a dynamic testing.

Verification	Validation
It comprises checking documents, design, codes and programs.	It comprises testing and validating the actual application.
It is a static testing.	It is a dynamic testing.
It does not comprise of the execution of the developed codes.	It certainly comprises of the execution of the developed code.
Approaches used in this case are reviews, walkthroughs, inspections and desk-checking.	Approaches used in this case are Black Box Testing, White Box Testing and non-functional testing.
It checks whether the developed application conforms to specifications or not.	It checks whether the developed application addresses the necessities and expectancies of a client or not.
Using this process, program bugs can be identified at the early stage of the development.	It can only find the program bugs that was not identified by the verification process.
The purpose of verification is application and software architecture and specification.	The purpose of validation is on the final application.
Verification is accomplished by the Quality assurance team.	Testing team is responsible for validation related activities on software codes.
This process comes prior to validation.	This process comes post verification.
It involves checking of documents/files and is performed by a human being.	It involves execution of programs and is performed by the system (computer) itself.

Model Calibration Techniques

The following three steps are used to yield precise simulation models. They are:

- Calibration
- Verification
- Validation

Computer simulations are certainly excellent means of illustrating and performing comparison of theoretical situations but in case to precisely model live cases, there has to be a harmonization with the live system which is under production. In such case and in ideal scenario, a base model needs to be developed and calibrated so that the model is capable of matching that specific extent which is under the purview of study. The calibrated model should then be tested to guarantee that the model is functioning as anticipated on the basis of real inputs. Post the model is confirmed, the concluding step is to confirm the functioning of the model by comparing the outputs with the data taken from the real live system. This can be performed by using various statistical methodologies and guaranteeing a satisfactory R-squared denomination. Until and unless these methodologies are deployed, the simulation model developed shall yield imprecise outcomes and shall not be a beneficial forecasting instrument.

Model calibration is attained by regulating or altering any existing parameters to fine-tune how the model functions and simulates the procedure. For example, in auto-traffic simulation, distinctive parameters comprise look-ahead distance, car-following sensitivity, discharge headway and start-up lost time, which are by default considered to be standard parameters. These parameters bring out the vehicle driver's behaviours such as when and how long it will take the driver to change the associate lanes, how much space does a driver allows between itself and the vehicle in the front side and how fast the driver initiates to accelerate via an intersection. Regulating the said parameters has a straight consequence on the measure of traffic capacity that can cross via the modelled thoroughfare network by generating the vehicle drivers more or less forceful. These are some of the examples of calibration parameters which can be tweaked to equate with features experienced in the area at the study arena. Most of the traffic models shall obtain specific default values but there might be requirements of adjusting to for improved matches for the driver's behaviour at the arena which is under the purview of study.

Model verification process is attained by gaining output data from the specific developed model and comparing it to with the expected data from the input data. For example, in vehicle traffic simulation, volume of traffic can be substantiated to guarantee that real capacity quantity in the model is rationally adjacent to traffic capacity input into the model. Ideally, 10% shall be the threshold referred to in traffic simulation to regulate if output volumes are rationally adjacent to input capacity. Simulation models deal model inputs using various methodologies so traffic which makes entry in the network, i.e., might or might not attain its opted terminus. Furthermore, traffic which opts to make entry in the network might not be able to enter in case any bottleneck occurs. This is the reason the model verification is an essential aspect of the modelling procedure.

The concluding stage is to authenticate the model by comparing the consequences with what is anticipated based on past data from the area of study. Preferably, the model should generate analogous outcomes to what has taken place, historically. This is classically confirmed by means of referring to the R2 statistic from the fit. This statistic procedures the fraction of changeability that is taken into consideration by the developed specific model.

It is not at all mandatory that a high R2 value shall fit into the model well. Another tool is also used to confirm models and it is referred to as graphical

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residual analysis. In case a model's output values are radically diverse than the historical values, it undoubtedly signifies that there is certainly a fault in the model. This is a very significant stage to confirm prior to referring the model as a mean to yield supplementary models for diverse situations to guarantee each one is precise. If the yield outputs fail to rationally match historic values at the time of the validation process, the model should be immediately revised and restructured accordingly to yield outcomes more in line with accordance to the required or expected expectations. It is considered to be a reiterative procedure that assistances to yield further accurate models.

Three areas of erroneousness situations may be the primary reason of feeble correlation at the time of calibration. They are:

- Input error – Error occurred due to incorrect inputs
- Model error - Error occurred due to incorrect modelling structure
- Parameter error - Error occurred due to incorrect parameters

Normally, input error and parameter error can be attuned effortlessly by the user. Model error is generally caused due to the incorrect procedure used in the model. This can never be easily addressed. Simulation models are characteristically constructed by means of various different modelling philosophies and practices which can certainly yield contradictory outcomes. Some models are more comprehensive while others models are additionally exhaustive. As a result, erroneous models, it may require to fine-tune the model practice in order to make the outcomes additionally coherent.

To yield a decent model, which is used to generate accurate outcomes, above mentioned steps are the mandatory steps which needs to be followed. Simulation models should be used as an application to authenticate engineering theories but are solitarily effective in case it is calibrated in an appropriate manner. When acceptable approximations of the parameters for all models have been attained, the models should be examined to guarantee that they sufficiently achieve the functionality for which they are envisioned. The validation procedure ensures the trustworthiness of the model by indicating its capability to replicate the genuine traffic patterns. The significance of the model validation Notes underlines the requirement for vigilant planning, meticulousness and correctness of the input data collection sequencer that has the specified determination. Efforts should be given to guarantee the collected data is reliable with predictable values.

Independent Verification and Validation (IV&V) events take place during most of the systems engineering development life-cycle phases and are vigorously associated to them, as portrayed in the Figure 3.1, rather than being incomplete to integration and testing phases.

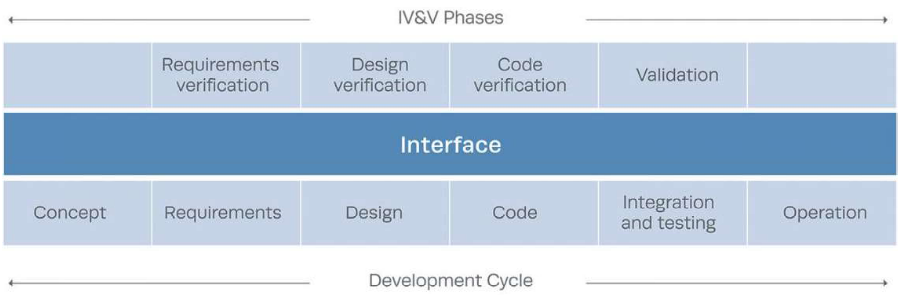

Fig. 3.1 *Verification and Validation of Simulation Models*

Various methodologies are used to validate the simulation models: They are provided with definitions in the following table:

Model Validation Method	Description
Comparison to with other Models	Numerous results (e.g., outputs) of the simulation model which are being validated are compared to with the results of other (effective) models. It can be something like a simulation model is compared to known results of a analytic models and the simulation model is compared to other simulation models that have already been validated.
Face Validity	This process consists of requesting people to provide their knowledge and feedback about the model and seeking their opinion about whether the behaviour of the model is ok or not. The correctness of logic of the conceptual model can also be enquired from the people. The input-output relationships related enquiry can also be done?
Historical Data Validation	In case the historical data exist, a portion of the data is utilized to form the model and the rest portion are utilized to identify (test) whether the model is behaving as the live system does.
Parameter Variability – Sensitivity Analysis	This method comprises of altering the values of the input and internal parameters of a model to find out the consequence of output on the model's behaviour. The same relations should take place in the model as well as in the real live system. This technique is normally used qualitatively i.e., instructions only of outputs and quantitatively i.e., both the directions and degrees of the outputs. Parameters that are delicate must be taken care of adequately and correctly preceding to using the model.
Predictive Validation	The model is used to forecast the system's behaviour. After that the system's behaviour and the corresponding model's predictions are compared to identify if they are the identical or not. It may happen that the system's data might come from an operational system or is collected by accompanying experimentations on the system.

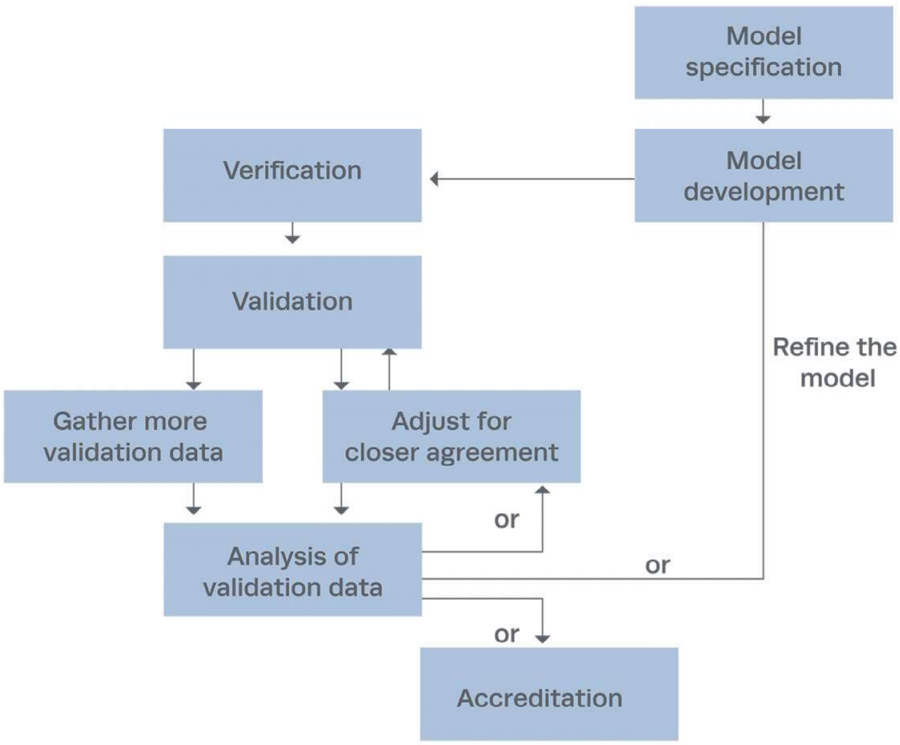

Fig. 3.2 *Simulation Model Development and the Verification, Validation and accreditation process (VV&A)*

Figure 3.2 portrays the relationship between model development and VV&A in its wholeness, it might also be applicable to discrete and autonomous model elements as well. A model VV&A strategy should recognize those model elements that might be authenticated autonomously of others. For those kinds of models, associated elements that establishes the interaction, the strategy should recognize suitable verification and validation methods.

3.3 ESTIMATION METHODS

- This method is used to estimate the range for the random variable to obtain the desired output.
- Infinite populace of data has a stationary probability distribution with a finite mean μ and finite variance σ^2.
- Sample variable and time do not disturb the population distribution.
- Central limit theorem should be taken into consideration to depend upon normal distribution of infinite population.
- Post this, we can only employ estimation method to that particular or specific variable extracted from the infinite population.
- A random variable is extracted from an infinite population that has a stationary probability distribution with a finite mean, μ, and finite variance, σ^2.
- Random variables which are able to satisfy all the said circumstances or conditions are known to be independently and identically distributed (i.i.d) and for this the central limit theorem is certainly applicable.
- The theorem states that the summation of n i i d variables extracted from a populace that has a mean of μ and a variance of σ^2, is nearly distributed as a standard variable with a mean of $n\mu$ and a variance of $n\sigma^2$.
- Let x_i (where i=1,2,…,n) be the n i.i.d. random variables. Then normal variate:

$$z = \frac{\sum_{i=1}^{n} x_i - n\mu}{\sqrt{n}\,\sigma}$$

- In terms of sample mean $\bar{x}$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

where

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- The probability density function of the standard normal variate is illustrated in the Figure Fig. 3.3.

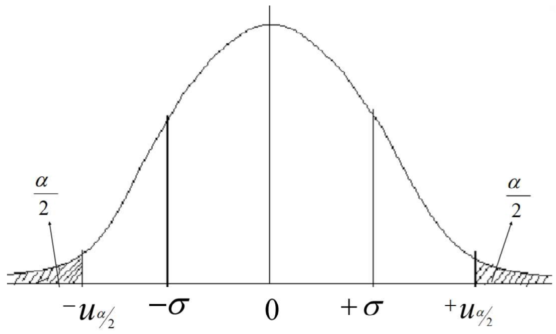

Fig. 3.3 *Probability Density Function of the Standard Normal Variate*

- The integral from -∞ to a value μ is the probability that Z is less than or equal to μ. The integral is denoted by
- Suppose the value of μ is chosen so that $\phi(u) = 1 - \alpha / 2$ where α is some constant less than 1, and denote this l value o f u by $u_{\alpha/2}$.
- The normal distribution is symmetric about its mean, so the probability that Z is less than $-u_{\alpha/2}$ is also α/2.
- The probability that z lies between $-u_{\alpha/2}$ and $u_{\alpha/2}$ is 1-α.

  That is,

$$\text{Prob}\{-u_{\alpha/2} \leq z \leq u_{\alpha/2}\} = 1 - \alpha$$

- With respect to sample mean, this probability statement can certainly be inscribed as:

$$\text{Prob}\{\bar{x} + \frac{\sigma}{\sqrt{n}} u_{\alpha/2} \geq \mu \geq \bar{x} - \frac{\sigma}{\sqrt{n}} u_{\alpha/2}\} = 1 - \alpha$$

- The constant 1-α is the confidence level and the confidence interval is as follows:

$$\bar{x} \pm \frac{\sigma}{\sqrt{n}} u_{\alpha/2}$$

- Typically, the confidence level might be 90% in which case $u_{\alpha/2}$ is 1.65.
- The population variance σ^2 is normally unknown; in such circumstance the population variance is substituted by an approximation which is calculated from the following formula.

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

- The normalized random variable based on σ^2 is substituted by a normalized random variable based on S^2. This has a Student-t distribution, with n-1 degrees of freedom.
- The quantity $u_{\alpha/2}$ used in the definition of a confidence interval, is substituted by alike quantity $t_{n-1,\alpha/2}$, which is based on the Student-t distribution.
- The Student -t distribution is severely precise only when the population from which the samples are extracted are generally distributed.

- Expressed in terms of the estimated variance S^2, the confidence interval for $\overline{x}$ is defined by the following equation:

$$\overline{x} \pm \frac{s}{\sqrt{n}} t_{n-1, \alpha/2}$$

- So, the estimation method yields the expected series of the sample variables extracted from infinite population.

Check Your Progress

1. What is verification?
2. Define the term validation.
3. What are the three steps to yield precise simulation model?
4. Why estimation method is used?

3.4 SIMULATION RUN STATISTICS

- This is a method which is normally used to address difficulties that ascend during the measurement of statistics from simulation run.
- To be more precise, it is a process to analyse the simulation result.
- There are two assumptions which are maintained to ascertain the self-assurance levels. They are:
  - Observations should be independent
  - Distribution should be stationary
- Let us take in account a single-server system in which the influxes take place with a Poisson distribution and the service time has an exponential distribution.
- In case the purpose of the investigation is to calculate the mean waiting time, expressed as the time entities devote waiting to obtain service and discounting the service time itself.
- This system is usually represented by M/M/1 which specifies;
  - First of all, the inter-arrival time is distributed exponentially
  - Secondly, the service time is distributed exponentially
  - Thirdly, there exists a single server only.

  M represents Markovian, which infers an exponential distribution.
- In a simulation run, the easiest method is to approximate the mean waiting time by accruing the waiting time of n successive entities and dividing it by n.
- This calculation which is known as sample mean, is signified by x(n) to establish the circumstance that its associated denomination is dependent on the occurrence of observations considered.
- If x_i (where i=1, 2… n) are the discrete waiting times (including the value 0 for those particular entities that is not waiting), then

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$$\bar{x}(n) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- Whenever a waiting line is established, the waiting time of each and every entity on the line distinctly is dependent on the waiting time of its predecessors.
- Any set of data which has the attribute of having one value affecting other values is said to be autocorrelated.
- The sample mean of autocorrelated data can be publicized to approximate a normal distribution with the increase of the size of the sample.
- The following equation stays acceptable for estimating the mean of autocorrelated data.

$$\bar{x}(n) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- A simulation run is initiated with the system at the given below states:
  - in a specific initial state
  - frequently during the idle state
  - during the time when no service/services is/are spawning
  - entities are not waiting
- The initial entrances then have a additional than usual probability of gaining service speedily, so a sample mean that comprises of the initial entrances will be biased.
- For a specific sample size initiating from a prearranged initial condition, the sample mean distribution is stationary, but, if the distributions could be contrasted for dissimilar sample sizes, the distribution shall be somewhat different.
- The below stated illustration (Fig. 3.4) is based on theoretical results. It portrays how the anticipated value of sample mean is dependent upon the sample length, i.e., for the M/M/1 system, initiating from a vacant state, with a server utilization of 0.9 which denoted by p.

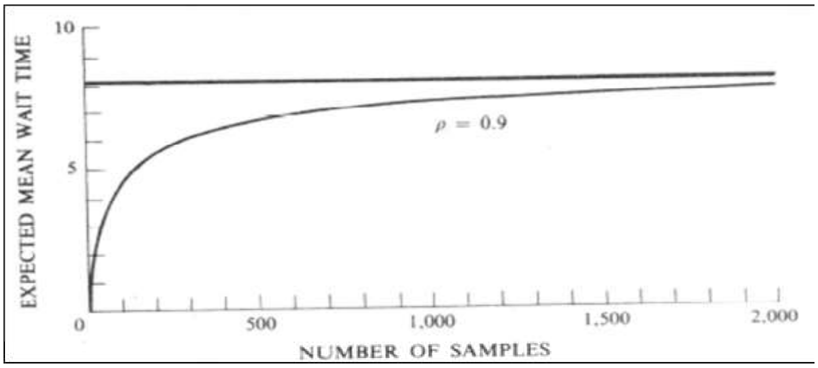

***Fig. 3.4** Mean Wait Time in M/M/1 System for different Sample Sizes*

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3.4.1 Replication of Runs

- The meticulousness of outcomes of a dynamic stochastic can be amplified by means of repetition of the experimentation with various random numbers strings.
- For every replication of a trivial sample extent, the sample mean is recognised.
- The sample means of the autonomous executions can be supplementary used to assess variance of the specific distribution. Let X_{ij} be the i^{th} observation in i^{th} run, then the sample mean and variance for the j^{th} run should be:

$$\overline{x_j}(n) = \frac{1}{n}\sum_{i=1}^{n} x_{ij}$$

$$s_j^2 = \frac{1}{n-1}\sum_{i=1}^{n}[x_{ij} - \overline{x_j}(n)]^2$$

- When we have analogous means and variances for m autonomous dimensions then by amalgamating them, the mean and variance for the population can be gained as follows:

$$\bar{x} = \frac{1}{p}\sum_{j=1}^{p}\overline{x_j}(n)$$

$$s^2 = \frac{1}{p}\sum_{j=1}^{p} s_j^2(n)$$

- The below provided illustration (Figure 3.5) depicts the consequence of applying the procedure to experimental consequences for the M/M/1 system.

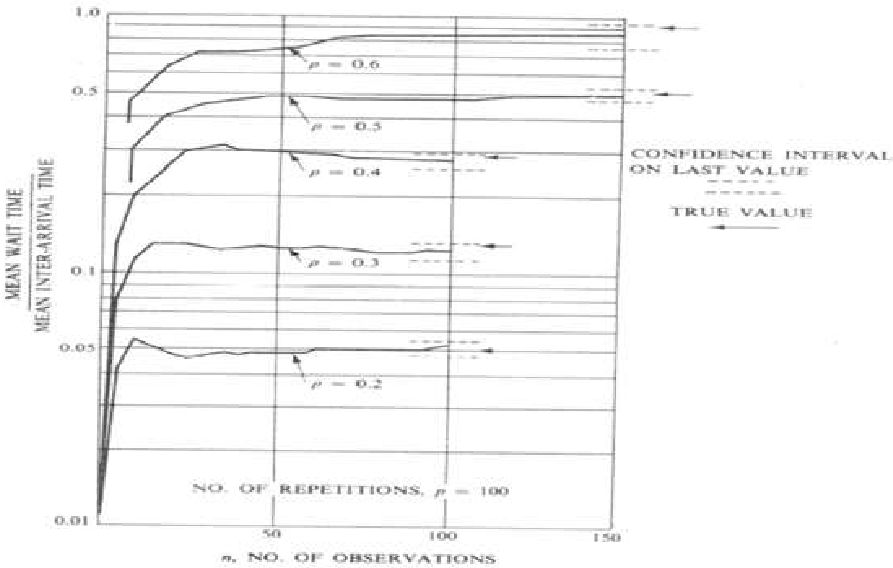

Fig. 3.5 *Analysis of Simulation*

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- This variance can subsequently be utilized to ascertain the confidence interval for p-1 degrees of freedom.
- The span of run of replications is chosen in such a way that all combination arrives to the size of the sample N i.e., p.n = N.
- In case, the number of replications is augmented and their length of run is reduced, the confidence interval shall be narrowed.
- But with the reducing of the length of replication, the effect of the initiating conditions shall upsurge.
- The outcomes gained may not be precise, particularly when the initiation of the runs is inappropriate.
- To address this hindrance, a compromise is essential means.
- There is no conventional technique of dividing the sample size N into replications.
- Though, it can be suggested that the number of replications should not be extremely hefty and that the sample means should ballpark a standard distribution.

The sequence of the run process is portrayed in Figure 3.6.

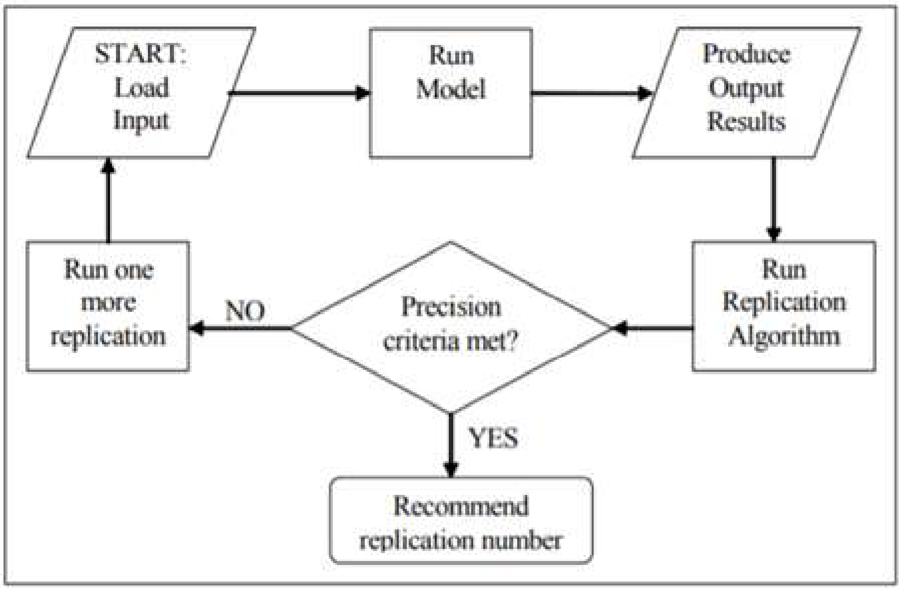

Fig. 3.6 *Flow Diagram of the Sequential Procedure*

The Replication Algorithm is portrayed in Figure 3.7.

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Let $n = 3$
Set $d_{required}$
Set *kLimit*

Run *n* replications of model.

While *Convergence Criteria* not met Do

- Calculate cumulative mean, ($\overline{X}_n$), confidence limits, and precision, (d_n).
- If $d_n \leq d_{required}$
  - ➢ Let *Nsol* be equal to *n*, the number of replications required to reach $d_{required.}$
  - ➢ If *kLimit* > 0
    - ▪ Calculate *f*(*kLimit*).
    - ▪ If the *Convergence Criteria* is met i.e. $d_{n+1}, \ldots, d_{n+f(kLimit)} \leq d_{required.}$
      Then
      Recommend *Nsol* replications to user
      Else
      Perform one more replication.

Loop

Fig. 3.7 *Replication Algorithm*

3.4.2 Regenerative Techniques

Regenerative processes were outlined and expressed by Walter L. Smith in Proceedings of the Royal Society in 1955.

In applied probability, a regenerative process is a type of stochastic process with a specific attribute that specific percentages of the process can be taken into consideration as being statistically self-reliant of each other. This attribute can be referred to in the derivation of theoretical attributes of such specific processes.

To be more precise, Regenerative simulation is defined by an assemblage of statistical procedures for scrutinising the outcome of a discrete-event stochastic simulation whose underlying stochastic process $\{X(t):t\ e \geq 0\}$

An example of regenerative process is a stochastic process. From a probabilistic point of the time is the main pointer here. The time point might be determined by the progression of the specific process. To be more precise, the process $\{X(t),\ t\ e \geq 0\}$ is considered to be a regenerative process if there is a presence of time points $0 \geq T0 < T1 < T2 < \ldots$ such that the post- T_k process $\{X(T_k + t) : t\ e \geq 0\}$

- has the same distribution as the post- T_0 process $\{X(T_0 + t) : t\ e \geq 0\}$
- is independent of the pre- T_k process $\{X(t) : 0\ d \geq t < T_k\}$

for $k\ e \geq 1$.

Intuitively is defined by a regenerative process which be fragmented into independent and identically distributed random variables cycles.

When $T_0 = 0$, X(t) is known as a nondelayed regenerative process. Otherwise, the process is known as a delayed regenerative process.

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The simulation of a stochastic system e.g., a queuing system is certainly a statistical phenomenon.

Check Your Progress

5. How will you define the simulation run statistics?
6. State the replication of runs.
7. When were regenerative techniques outlined?

3.5 ANSWERS TO 'CHECK YOUR PROGRESS'

1. Verification is the procedure of examination and scrutinizing that a software accomplishes its target without any errors. It is the procedure to guarantee whether the application which is developed is appropriate or not. It validates whether the developed application achieves the necessitates that has been defined as requirement. Verification is always a static testing.
2. Validation is the procedure of inspecting whether the developed application is up to the mark or not. It is the procedure of scrutinizing the validations of the developed application. In other words, it inspects whether the validations are able to produce the correct output as per expectations Validation is always a dynamic testing.
3. The three steps to yield precise simulation model are:
   (i) Calibration
   (ii) Verification
   (iii) Validation
4. Estimation method is used to estimate the range for the random variable to obtain the desired output.
5. This is a method which is normally used to address difficulties that ascend during the measurement of statistics from simulation run.
6. The meticulousness of outcomes of a dynamic stochastic can be amplified by means of repetition of the experimentation with various random numbers strings. For every replication of a trivial sample extent, the sample mean is recognised.
7. Regenerative processes were outlined and expressed by Walter L. Smith in Proceedings of the Royal Society in 1955.

3.6 SUMMARY

- Verification is the procedure of examination and scrutinizing that a software accomplishes its target without any errors. It is the procedure to guarantee

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whether the application which is developed is appropriate or not. It validates whether the developed application achieves the necessitates that has been defined as requirement. Verification is always a static testing.

- Validation is the procedure of inspecting whether the developed application is up to the mark or not. It is the procedure of scrutinizing the validations of the developed application. In other words, it inspects whether the validations are able to produce the correct output as per expectations Validation is always a dynamic testing.
- Estimation method is used to estimate the range for the random variable to obtain the desired output.
- Simulation run statistics is a method which is normally used to address difficulties that ascend during the measurement of statistics from simulation run.
- The meticulousness of outcomes of a dynamic stochastic can be amplified by means of repetition of the experimentation with various random numbers strings. For every replication of a trivial sample extent, the sample mean is recognised.
- In applied probability, a regenerative process is a type of stochastic process with a specific attribute that specific percentages of the process can be taken into consideration as being statistically self-reliant of each other.

3.7 KEY TERMS

- **Verification:** Verification is the procedure of examination and scrutinizing that a software accomplishes its target without any errors. It is the procedure to guarantee whether the application which is developed is appropriate or not. It validates whether the developed application achieves the necessitates that has been defined as requirement. Verification is always a static testing.
- **Validation:** Validation is the procedure of inspecting whether the developed application is up to the mark or not. It is the procedure of scrutinizing the validations of the developed application. In other words, it inspects whether the validations are able to produce the correct output as per expectations Validation is always a dynamic testing.

3.8 SELF-ASSESSMENT QUESTIONS AND EXERCISES

Short-Answer Questions

1. What do you understand by the validation and verification?
2. Learn about the estimation method.
3. What is simulation run statistics?
4. State about the replication of runs.
5. How will you define the regenerative techniques?

Long-Answer Questions

1. Differentiate between the validation and verification with the help of examples.
2. Explain briefly about the estimation method. Give appropriate examples.
3. Briefly explain the simulation run statistics with the help of examples.
4. Discuss about the replications of runs. Give appropriate examples.
5. Illustrate the regenerative techniques with the help of examples.

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3.9 FURTHER READING

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UNIT 4 MONTE CARLO METHODS

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Structure

4.0 INTRODUCTION

In this unit, you will learn about simulation. Simulation is a representation of reality through the use of a model or any other device which will react like a real one under a given set of conditions. Simulation can be analog or digital. Digital simulation is also known as computer simulation. You will understand the importance and application of simulation techniques. Simulation is the process of designing a mathematical or logical model of a real system and then accomplishing experiments with the model to describe, explain and predict the behavior of the real system. A static simulation model represents a system at a particular point in time whereas a dynamic simulation model represents a system developed over time. Based on these two classifications, a simulation may be classified as deterministic or stochastic. A deterministic simulation model is one that contains no random variables whereas a stochastic simulation model contains one or more random variables. You will also learn about multiplicative congruential algorithm and inverse transformation method. The Monte Carlo simulation model is used by modern management. It uses random number tables to reproduce on paper the operation of any given system under its own working conditions. You will also learn about the three models of Monte Carlo simulation used in three different contexts. These are queuing theory, inventory control and production line. The unit will familiarize you with the Bayes' theorem as well. Finally, the unit will discuss random variable and probability distribution functions in detail.

4.1 OBJECTIVES

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After going through this unit, you will be able to:

- Understand the basic concepts of simulation
- Analyse the applications and importance of simulation
- Discuss the simulation of a queueing system using event list
- Describe the significance of multiplicative congruential algorithm and inverse transformation method
- Understand Monte Carlo simulation and its application
- Explain the three context models of Monte Carlo simulation
- Understand Bayes' theorem
- Know about random variable and probability distribution functions

4.2 SIMULATION AND RANDOM VARIABLE: BASIC CONCEPTS

Simulation is the most widely used flexible modelling approach. This approach is used to model the behaviour of individual components within the system with the help of random sampling technique for generating pragmatic variability. The simulation process includes problem definition, conceptual modelling, model coding, model verification and validation, experimentation and analysis of results, and solution implementation.

4.2.1 Types of Simulation

Simulation is mainly of two types:

(*i*) Analog (environmental) simulation.

(*ii*) Computer (system) simulation.

Some examples of simulation models are given below:

(*i*) Testing an aircraft model in a wind tunnel.

(*ii*) Children cycling in a park with various signals and crossings—to model a traffic system.

(*iii*) Planetarium.

To determine the behaviour of a real system in actual environment, a number of experiments are performed on simulated models either in the laboratories or on the computer itself.

4.2.2 Random Variable

The random variable is a real-valued function, defined over a sample space associated with the outcome of a *conceptual* chance experiment. Random variables are classified according to their probability density function.

Random Number: It refers to a uniform random variable or a numerical value assigned to a random variable, following uniform probability density function. In other words, it is a number in a sequence of numbers, whose probability of occurrence is the same as that of any other number in that sequence.

Pseudorandom Numbers: Random numbers are called *pseudorandom numbers* when they are generated by some deterministic process, but have already qualified the pre-determined statistical test for randomness.

4.2.3 Monte Carlo Technique or Monte Carlo Simulation

The Monte Carlo method is a simulation technique in which statistical distribution functions are created by using a series of random numbers. The method is generally used to solve the problems that cannot be adequately represented by the mathematical models or where the solution of the model cannot be arrived at, by analytical method.

Monte Carlo simulation yields a solution very close to the optimal solution, but not necessarily the exact solution. The Monte Carlo simulation procedure can be summarized in the following six steps:

Step 1: Clearly define the problem.

(*a*) Identify the objectives of the problem.

(*b*) Identify the main factors which have the greatest effect on the objectives of the problem.

Step 2: Construct an appropriate model.

(*a*) Specify the variables and parameters of the model.

(*b*) State the conditions under which the experiment is to be performed.

(*c*) Define the relationship between the variables and parameters.

Step 3: Prepare the model for experimentation.

(*a*) Define the starting conditions for the simulation.

(*b*) Specify the number of runs of simulation to be made.

Step 4: Using Steps 1 to 3, experiment with the model.

(*a*) Define a coding system that will correlate the factors defined in Step 1 with the random numbers to be generated for the simulation.

(*b*) Select a random number generator and create the random numbers to be used in the simulation.

(*c*) Associate the generated random numbers with the factors identified in Step 1 and coded in Step 4 (*a*).

Step 5: Summarize and examine the results obtained in Step 4.

Step 6: Evaluate the results of the simulation.

4.2.4 Generation of Random Numbers

Monte Carlo simulation needs the generation of a sequence of random numbers, which constitute an integral part of the simulation model and also help in determining random observations from the probability distribution.

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Random numbers may be found through a computer using random tables or manually. The most common method to obtain random numbers is to generate them through a computer program.

Example 4.1: A sample of 100 arrivals of a customer at a retail sales depot is according to the following distribution:

Time between Arrival (min.)	*Frequency*
0.5	2
1.0	6
1.5	10
2.0	25
2.5	20
3.0	14
3.5	10
4.0	7
4.5	4
5.0	2

A study of the time required to service customers by adding up the bills, receiving payments and placing packages, yields the following distribution:

Time between Arrival (min.)	*Frequency*
0.5	12
1.0	21
1.5	36
2.0	19
2.5	7
3.0	5

Estimate the average percentage of customer waiting time and average percentage of idle time of the server, by simulation, for the next 10 arrivals.

Solution: Tag numbers are allocated to the events in the same proportions as indicated by the probabilities.

Arrivals	*Frequency*	*Probability*	*Cumulative Probability*	*Tag Numbers*
0.5	2	0.02	0.02	00–01
1.0	6	0.06	0.08	02–07
1.5	10	0.10	0.18	08–17
2.0	25	0.25	0.43	18–42
2.5	20	0.20	0.63	43–62
3.0	14	0.14	0.77	63–76
3.5	10	0.10	0.87	77–86
4.0	7	0.07	0.94	87–93
4.5	4	0.04	0.98	94–97
5.0	2	0.02	1.00	98–99

Service Time (min.)	Frequency	Probability	Cumulative Probability	Tag Numbers
0.5	12	0.12	0.12	00–11
1.0	21	0.21	0.33	12–32
1.5	36	0.36	0.69	33–68
2.0	19	0.19	0.88	69–87
2.5	7	0.07	0.95	88–94
3.0	5	0.05	1.00	95–99

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The random numbers are generated and linked to the appropriate events. The first 10 random numbers, simulating arrival, the second 10, simulating service times. The results are incorporated in Table 4.1, on the assumption that the system starts at 0.00 a.m.

Average waiting time per customer is, $\frac{4.5}{10} = 0.45$ minutes.

Average waiting time (or) idle time of the servers = $\frac{7.00}{10} = 0.7$ minutes.

Table 4.1

Arrival No.	Random Number	Inter-Arrival Time (min.)	Arrival Time (min.) *a*	Random No.	Service Time (Min.) *e*	Service Start *b*	End *f*	Waiting Time of Customer *c = b – a*	Waiting Time of Server *d = f – a*
1	93	4.0	4.0	78	2.0	4	6	–	4.0
2	22	2.0	6.0	76	2.0	6	8	–	–
3	53	2.5	8.5	58	1.5	8.5	10.0	–	0.5
4	64	3.0	11.5	54	1.5	11.5	13	–	1.5
5	39	2.0	13.5	74	2.0	13.5	15.5	–	0.5
6	07	1.0	14.5	92	2.5	15.5	18	1.0	–
7	10	1.5	16.0	38	1.5	18.0	19.5	2.0	–
8	63	3.0	19.0	70	2.0	19.5	21.5	0.5	–
9	76	3.0	22.0	96	3.0	22.0	25.0	–	0.5
10	35	2.0	24.0	92	2.5	25.0	27.5	1.0	–
							Total	4.5	7.0

Example 4.2: A tourist car operator finds that during the past few months, the car's use has varied so much that the cost of maintaining the car varied considerably. During the past 200 days the demand for the car fluctuated as below.

Trips Per Week	Frequency
0	16
1	24
2	30
3	60
4	40
5	30

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Using random numbers, simulate the demand for a 10-week period.

Trips/Week or Demand/Week	*Frequency*	*Probability*	*Cumulative Probability*	*Tag Numbers*
0	16	0.08	0.08	00–07
1	24	0.12	0.20	08–19
2	30	0.15	0.35	20–34
3	60	0.30	0.65	35–64
4	40	0.20	0.85	65–84
5	30	0.15	1.00	85–99

Solution: The tag numbers allotted for the various demand levels are shown in the table above.

Week	*Random Number*	*Demand*
1	82	4
2	96	5
3	18	1
4	96	5
5	20	2
6	84	4
7	56	3
8	11	1
9	52	3
10	03	0
		Total = 28

The simulated demand for the cars for the next 10 weeks period is given in the table above.

Total demand = 28 cars.

Average demand = $\frac{28}{10}$ = 2.8 cars per week.

Example 4.3: A manufacturing company keeps stock of a special product. Previous experience indicates the daily demand as given below.

Daily Demand	*5*	*10*	*15*	*20*	*25*	*30*
Probability	0.01	0.20	0.15	0.50	0.12	0.02

Simulate the demand for the next 10 days. Also find the daily average demand for the product on the basis of simulated data.

Solution: The solution can be obtained as follows:

Demand	*Probability*	*Cumulative Probability*	*Tag Numbers*
5	0.01	0.01	00–00
10	0.20	0.21	01–20
15	0.15	0.36	21–35
20	0.50	0.86	36–85
25	0.12	0.98	86–97
30	0.02	1.00	98–99

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Day	*Random Number*	*Demand*
1	82	20
2	96	25
3	18	10
4	96	25
5	20	10
6	84	20
7	56	20
8	11	10
9	52	20
10	03	10
		Total = 170

$$\therefore \qquad \text{Average demand} = \frac{170}{10} = 17 \text{ units/day.}$$

Example 4.4: An automobile production line turns out about 100 cars a day but deviations occur owing to many causes. The production is more accurately described by the probability distribution given below.

Production/Day	*Probability*	*Production/Day*	*Probability*
95	0.03	101	0.15
96	0.05	102	0.10
97	0.07	103	0.07
98	0.10	104	0.05
99	0.15	105	0.03
100	0.20	106	0.60
			1.00

Finished cars are transported across the bay at the end of each day by a ferry. If the ferry has space for only 101 cars, what will be the average number of cars waiting to be shipped and what will be the average number of empty space on the ship?

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Solution: The tag numbers are established in the table below.

Production/Day	*Probability*	*Cumulative Probability*	*Tag Numbers*
95	0.03	0.03	00–02
96	0.05	0.08	03–07
97	0.07	0.15	08–14
98	0.10	0.25	15–24
99	0.15	0.40	25–39
100	0.20	0.60	40–59
101	0.15	0.75	60–74
102	0.10	0.85	75–84
103	0.07	0.92	85–91
104	0.05	0.97	92–96
105	0.03	1.00	97–99

The simulated production of cars for the next 15 days is given in the following table.

Day	*Random Number*	*Production Per Day*	*No. of Cars Waiting*	*No. of Empty Space in the Ship*
1	97	105	4	–
2	02	95	–	6
3	80	102	1	–
4	66	101	–	–
5	96	104	3	–
6	55	100	–	1
7	50	100	–	1
8	29	99	–	2
9	58	100	–	1
10	51	100	–	1
11	04	96	–	5
12	86	103	2	–
13	24	98	–	3
14	39	99	–	2
15	47	100	–	1
		Total	10	23

Average number of cars waiting to be shipped $= \frac{10}{15} = 0.67$ per day.

Average number of empty spaces on the ship $= \frac{23}{15} = 1.53$ per day.

Example 4.5: Suppose that the sales of a particular item per day is Poisson with mean five, then generate 20 days of sales by the Monte Carlo method.

Solution: The probability for the sales is given by,

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-5}5^r}{r!} \quad (\because \lambda = 5)$$

γ	*Cumulative Probability*	*Tag Numbers*
0	0.01	00–00
1	0.04	01–03
2	0.13	04–12
3	0.27	13–26
4	0.44	27–43
5	0.62	44–61
6	0.76	62–75
7	0.87	76–86
8	0.93	87–92
9	0.97	93–96
10	0.98	97–97
11	0.99	98–98
12	1.00	99–99

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The simulated sales for the next 20 days is given in the table below.

Day	*Random Number*	*Sales*	*Day*	*Random Number*	*Sales*
1	49	05	11	99	12
2	55	05	12	89	08
3	89	08	13	10	02
4	15	03	14	27	04
5	12	02	15	50	05
6	94	09	16	93	09
7	85	07	17	92	08
8	34	04	18	57	05
9	07	02	19	50	05
10	53	05	20	78	07

4.3 SIMULATION OF A QUEUEING SYSTEM USING EVENT LIST

Simulation is referred as a powerful and extensively used management science technique for the analysis and study of complex systems. Simulation technique is specifically used to imitate the function of a real world system that develops over time. Usually this is performed by developing a simulation model. As a rule, the simulation model typically obtains the form of a set of assumptions with reference to the function of the system and is expressed as mathematical or logical relations between the significant/concerned objects in the system.

Thus, simulation is the process of designing a mathematical or logical model of a real system and then accomplishing experiments with the model to describe, explain and predict the behavior of the real system.

Therefore, to model a system the basic concept of a system must be clear. By definition, a system is a collection of entities that act and interact toward the accomplishment of some logical end. Generally, the systems are referred to be dynamic as their status changes over time. Hence, to explain this status the concept

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of the state of a system is used. Simulation is very useful in understanding the expected performance of the real system and also in testing the efficiency of the system design. A static simulation model represents a system at a particular point in time whereas a dynamic simulation model represents a system developed over time.

Based on these two classifications, a simulation may be classified as deterministic or stochastic. A deterministic simulation model is one that contains no random variables whereas a stochastic simulation model contains one or more random variables.

An event is characteristically defined as a situation that causes the state of the system to change instantaneously. When the state of system transforms/changes only at discrete points in time termed as 'discrete events'. When the state of system changes continuously over time, then it is termed as 'continuous event'. All the information regarding change of state is preserved in a list called the event list. In a simulation, time is maintained using a variable termed as the clock time. This simulation is initiated with an empty system and arbitrarily assumed that the first event is an arrival that occurs at clock time 0. Subsequently the departure time of the first customer is scheduled.

Departure Time = Clock Time Now + Generated Service Time

Next step is to schedule the next arrival into the system by randomly generating an inter-arrival time from the inter-arrival time distribution and setting the arrival time as follows:

Arrival Time = Clock Time Now + Generated Inter-Arrival Time

Both these events have their own specific scheduled times which are maintained on the event list. In a system when events occur, time is advanced from event to event. This approach of simulation is termed as the next-event time-advance mechanism, because of the manner the clock time is updated. The simulation clock can be advanced to the time of the most imminent event.

Moving from event to event, the appropriate actions for each event is carried out including any scheduling of future events. The jump to the next event in the next-event mechanism may be a large one or a small one which specifies that the size of jumps can be changeable or varied. This approach can be distinguished with the fixed-increment time-advance method. With this method, the simulation clock is advanced in increments of Δt time units, where Δt is some appropriate time unit, usually 1 time unit.

However, for most of the models the next-event mechanism can be used to develop models. To demonstrate the computerized simulation model, the following variables must be defined:

TM = Clock time of the simulation.

AT = Scheduled time of the next arrival.

DT = Scheduled time of the next departure.

SS = Status of the server (1 = Busy, 0 = Idle).

WL = Length of the waiting line.

MX = Length (in time units) of a simulation run.

The simulation can be initiated by initializing all the variables to analyse a specific problem.

Event Scheduling: Time is advanced from event to event. The future events list illustrates the ordered list of upcoming events. The moment events are scheduled, they are added to the event list. Similarly, when events occur they are removed from event list.

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4.4 MULTIPLICATIVE CONGRUENTIAL ALGORITHM

A Linear Congruential Generator (LCG) represents one of the oldest and well known pseudorandom number generator algorithms. The generator is defined by the recurrence relation:

$$X_{n+1} \equiv (aX_n + c) \pmod{m}$$

Where,

X_n = The sequence of pseudorandom values.

$m, 0 < m$ = The modulus.

$a, 0 < a < m$ = The multiplier.

$c, 0 \leq c < m$ = The increment.

$X_0, 0 \leq X_0 < m$ = The start value or seed.

All these are integer constants that specify the generator. If $c = 0$, then the generator is called a *multiplicative congruential method* or Lehmer RNG. If $c \neq 0$, then the generator is called a *mixed congruential method*.

If $c = 0$, then we have a **power residue** or **multiplicative generator**.

Note that $Z_n = (aZ_{n-1}) \bmod(m) \Rightarrow Z_n = (a^n Z_0) \bmod(m)$.

If $m = 2^B$, where B = # bits in the machine, then the longest period is $m/4$ if and only if:

- Z_0 is odd.
- $a = 8k + 3,\ k \in Z^+$ (5,11,13,19,21,27,…)

Check Your Progress

1. How random variables are classified?
2. When the random numbers are called as pseudorandom numbers?
3. Why Monte Carlo simulation needs the generation of a sequence of random numbers?
4. Why simulation technique is used?
5. What are discrete and continuous events?
6. What does Linear Congruential Generator (LCG) represent?

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4.5 INVERSE TRANSFORMATION METHOD

In simulation process the inverse transformation method can be described on the basis of following two postulates.

Inverse Transform Method for Simulating Continuous Random Variables

Let X be a random variable with continuous decreasing function $F_X(x)$. Since $F_X(x)$ is a non-decreasing function, the inverse function $F_X^{-1}(y)$ may be defined for any value of y between 0 and 1 as follows:

$$F_X^{-1}(y) = inf\{ x: F_X(x) >= y\} \qquad 0<= y <= 1$$

$F_X^{-1}(y)$ is defined to equal that value x for which $F(x) = y$.

Theorem 4.1. If U is uniformly distributed over the interval (0, 1), then $X = F_X^{-1}(U)$ has continuous decreasing function $F_X(x)$.

Proof: We have,

$$P(X <= x) = P(F_X^{-1}(U) <= x) = P(U <= F_X(x)) = F_X(x).$$

So to get a value, say x, of a random variable X, obtain a value, say u, of a random variable U, compute $F_X^{-1}(U)$, and set it equal to x.

Example 4.6: Generate a random variable from the uniform distribution $U(a, b)$:

$$F_x(x) = \begin{matrix} 1/(b-a) & a \Leftarrow x \Leftarrow b \\ 0 & otherwise \end{matrix}$$

Solution: The continuous decreasing function is,

$$F(x) = \begin{pmatrix} 0 & x < a \\ \dfrac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{pmatrix}$$

$$U = \frac{x-a}{b-a}$$

$$X = F^{-1}x(U) = a + (b-a)U$$

Inverse Transform Method for Simulating Discrete Random Variables

The inverse transform method for simulating from continuous random variables have analog in the discrete case. For example, to simulate a random variable X having partial differential function,

$$P(X = x_j) = P_j \qquad j = 0, 1, \ldots$$

$$\sum_j P_j = 1$$

$$F(X_j) = \sum_{i=0}^{j} P_i$$

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To simulate X for which $P(X = x_j) = P_j$, consider that U be uniformly distributed over (0, 1) and set as,

$$X = \begin{pmatrix} x_0 & U < P_0 \\ x_1 & P_0 \le U < P_0 + P_1 \\ \cdots & \cdots \\ x_j & \sum_{i=0}^{j=1} P_i \le U < \sum_{i=0}^{j} P_i \end{pmatrix}$$

Since,

$$P(X = x_j) = P\left(\sum_{i=0}^{j=1} P_i \le U \le \sum_{i=0}^{j} P_i\right) = \int_{F(x_{j-1})}^{F(X_j)} dx = F(X_j) - F(X_{j-1}) = P_j$$

In this case X has the desired distribution.

Example 4.7: Simulating a Poisson random variable.

Solution: The random variable X is Poisson with mean λ if,

$$p_i = P(X = i) = e^{-\lambda}\frac{\lambda^i}{i!} \quad \text{where, } i = 0, 1, \ldots$$

The key to use the inverse transform method is to generate such a random variable with the following identity:

$$p_{i+1} = \frac{\lambda}{i+1} p_i, \quad \text{where, i>=0}$$

Using this recursion to compute the Poisson probabilities, the inverse transform algorithm for generating a Poisson random variable with the mean λ can be expressed as follows.

The quantity i refers to the value presently under consideration; $p = p_i$ is the probability that X is equal to i, and $F = F(i)$ is the probability that X is less than or equal to (i). Now follow the given steps:

Step 1: Generate a random number U.

Step 2: Let $i = 0$, $p = e^{-\lambda}$, $F = p$.

Step 3: If $U < F$, Set $X = i$ and Stop.

Step 4: Let $p = \lambda p/(I + 1)$, $F = F + p$, $i = i + 1$.

Step 5: Go to Step 3.

This algorithm generates a Poisson random variable with the mean λ. Note that it first generates a random number U and then checks whether or not $U < e^{-\lambda} = p_0$. If so, it sets $X = 0$. If not, then it computes p_1 by using the recursion method. It now checks whether $U < p_0 + p_1$ and if so it sets $X = 1$, and so on.

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Thus, the algorithm successively checks whether the Poisson value is 0, then whether it is 1, then 2, and so on. The number of comparisons required will be 1 greater than the generated value of the Poisson. Hence, the above algorithm will have $1 + \lambda$ searches. It now generates a Poisson random variable with the mean λ by generating a random number U.

4.6 BASIC IDEAS OF MONTE-CARLO SIMULATION

Monte Carlo simulation is often used by modern management when it cannot use other techniques. There are many industrial problems which defy mathematical solutions. The reason is that either they are too complicated or that the data cannot be expressed in mathematical terms. In such cases, it is still possible to reach valid conclusions by using the Monte Carlo technique. A considerable help is thus obtained at practically no cost in taking decisions concerning the functioning of a business system. The data and conclusions can be obtained through simulation of an actual operation on the basis of its own past working. It paves the way for predicting the changes in its behaviour and the result is evaluated from innovations that we want to introduce.

By using a fresh series of random numbers at the appropriate junctures we can also examine the reactions of the simulated model just as if the same alterations had actually been made in the system itself. Monte Carlo simulation, therefore, provides a tool of knowing in advance whether or not the expense to be incurred or the investment to be made in making the changes envisaged. Through this technique, you can introduce the innovations on a piece of paper, examine their effects and then may decide to adopt or not to adopt such innovations in the functioning of real system. The usefulness of simulation lies in the fact that it allows us to experiment with a model of the system rather than the actual system; in case we are convinced about the results of our experiments we can put the same into practice. Thus the effect of the actual decisions are tested in advance through the technique of simulation by resorting to the study of the model representing the real life situation or the system.

The main purpose of simulation in management is to provide feedback, which is vital for the learning process. It creates an atmosphere in which managers play a dynamic role by enriching their experience through involvement in reckoning with actual conditions through experimentation on paper. The technique permits trying out several alternatives as the entire production for service process can be worked out on paper, without dislocating the system in any way. Thus, Monte Carlo technique transforms the manager from a blind-folded driver of an automobile, reacting to instructions of a fellow passenger to one who can see fairly, clearly, where he is going.

Introduction to Simulation

Simulation is a representation of reality through the use of a model or other device which will react in the same manner as reality under a given set of conditions.

Simulation is also defined as the use of a system model that has the designed characteristics of reality in order to produce the essence of actual operation.

Table 4.2 is an example of simulation worksheet.

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Table 4.2 *Simulation Worksheet for Simulating Sizes to Locate the Number of Misfits*

Assembly $X = \mu+z(\sigma)$	*Shafts Random[1] Numbers (from S.N.*	*Random[2] Normal Deviate Tippett Tables)*	*Simulating[3] Size (z)*	*Rings Random Numbers (from Tippett* $X = \mu+z(\sigma)$	*Random Normal Deviate Tables)*	*Simulating Size (z)*
1	2952	0.82	0.980 + 0.82(0.01) = 0.9882	3992	1.28	1.0 + 1.28(0.02) = 1.0256
2	3170	0.91	0.980 + 0.91(0.01) = 0.9891	4167	1.38	1.0 + 1.38(0.02) = 1.0276
3	7203	– 0.59	0.980 – 0.59(0.01) = 0.9741	1300	0.33	1.0 + 0.33(0.02) = 1.0066
4	3408	1.00	0.980 + 1.0(0.01) = 0.9900	3563	1.06	1.0 + 1.06(0.02) = 1.0212
5	0560	0.14	0.980 + 0.14(0.01) = 0.981.4	1112	0.28	1.0 + 0.28(0.02) = 1.0056
6	6641	– 0.42	0.980 – 0.42(0.01) = 0.9758	9792	– 2.04	1.0 – 2.04(0.02) = 0.9592
7	5624	– 0.16	0.980 – 0.16(0.01) = 0.9784	9525	– 1.67	1.0 – 1.67(0.02) = 0.9666
8	5356	– 0.09	0.980 – 0.09(0.01) = 0.9791	2693	0.74	1.0 + 0.74(0.02) = 1.0148
9	2769	0.76	0.980 + 0.76(0.01) = 0.9876	6107	– 0.28	1.0 – 28(0.02) = 0.9944
10	5246	– 0.06	0.980 – 0.06(0.01) = 0.9794	9025	– 1.29	1.0 – 1.29(0.02) = 0.9742

Monte Carlo Method

Monte Carlo methods are basically the algorithms used in the computation of result to be calculated from repeated random sampling. These methods help in computerized calculations because these can perform repeated computation using random or pseudorandom numbers. It is also used when it is not feasible to compute correct result with a deterministic algorithm. Monte Carlo simulation methods are used to study systems having degrees of freedom and in the situations when there is significant ambiguity in inputs for example, calculating risk factor in a business.

Various simulation models, based on the principle of similitude (such as model of aeroplanes initiating flight conditions in a wind tunnel) have been in use for a long time. However, Monte Carlo simulation is a recent operations research innovation. The novelty lies in making use of pure chance to contact a simulated version of the process under analysis, in exactly the same way as pure chance

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operates the original system under working conditions. Only models under uncertainty can be evaluated using Monte Carlo technique.

'Monte Carlo' is the code name given by John von Neumann and S.M. Ulam to the technique of solving problems though it is too expensive for experimental solutions and too complicated for analytical treatment.

Monte Carlo method is not one single method. It involves various widely-used classes of approaches to follow a specific model. Using it the following can be done:

- Define a domain with feasible inputs.
- Randomly generate inputs from the domain.
- Perform deterministic computation with the inputs.
- Combine the results of the personal computations into the final result.

Monte Carlo methods are used to solve various mathematical problems based on sampling experiments in statistics using the sequences of random numbers for simulation and are termed as statistical simulation methods. Thus, Monte Carlo method is not one single method, but it is a collection of various methods and is basically used to perform similar procedure. Some of these methods are discussed here with the help of solved examples.

The Monte Carlo simulation technique can as well be used to solve probabilistic problems. Suppose, we are to evaluate the probability P that a tank will be knocked out by either a first or second shot from an antitank gun assumed to posses a constant kill probability of 1/2. The probability analysis will say that the chance of tank being knocked out by either a first or second shot from an antitank gun is $1/2+1/2\ (1-1/2) = 3/4$. However, we can also work out this probability by simulating each round of the antitank gun by the flip of a coin through Monte Carlo simulation technique.

Since the probability of a 'head' is the same as that of a kill, we may call it a hit when the coin turns up a head and otherwise a miss. If we flip the coin a large number of times, the value of P may be calculated by merely counting the number of times a head turns up at least in two successive throws and then dividing this number by the total pairs of throws of the coin. Monte Carlo method in this simple case is indeed a poor substitute for the theoretical probability analysis. However, many real-life systems are so complicated that even the well defined probability analysis very often fails but such situations can be handled by Monte Carlo simulation, particularly the Monte Carlo technique that provides the simplest possible solutions for queuing problems. Problems of corporate planning, inventory control, capital investment, consumer behaviour and quality control can also be handled through simulation.

Monte Carlo simulation uses random number tables to reproduce on paper the operation of any given system under its own working conditions. This technique is used to solve problems that depend upon probability where formulation of mathematical model is not possible. It involves first, the determining of the probability distribution of the concerned variables and then sampling from this distribution by means of random numbers to obtain data. It may, however, be emphasized here that the probability distributions to be used should closely resemble the real world situation.

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One should always remember that simulation is not a perfect substitute but rather an alternative procedure for evaluating a model. Analytical solution produces the optimal answer to a given problem, while Monte Carlo simulation yields a solution which should be very close to the optimal but not necessarily the exact correct solution. Monte Carlo Simulation solution converges to the optimal solution as the number of simulated trials goes to infinity.

4.7 APPLICATIONS OF SIMULATION TECHNIQUE

Monte Carlo simulation has been applied to a wide diversity of problems ranging from queuing process, inventory problem, risk analysis concerning a major capital investment, such as the introduction of a new product, expansion of the capacity, and many other problems. Budgeting is another area where simulation can be very useful. In fact, the system of flexible budgeting is an exercise in simulation. Simulation can as well be used for preparing the master budget through functional budgets.

Over and above, the greatest contribution of simulation is in the analysis of complex systems. Many real-world problems involve systems made up of many components parts that are interrelated. The system may be dynamic and changing over time and may involve probabilistic or uncertain events. Simulation is the only technique for quantitative analysis of such problems.

Monte Carlo Simulation in the Context of Queuing Theory

Monte Carlo simulation which uses random number tables can better be illustrated by considering any concrete operation subject to chance. Let us take the arrival of scooters at a service station. First of all, observe the actual arrivals of scooters on number of days, say for five days, then put this information in the following two ways:

- Group the number of scooters arriving every hour, say between 7–8, 8–9 a.m., and so on till 4–5 p.m. (assuming the working hours of the service station is from 7 a.m. to 5 p.m. and also assuming that no scooter arrives before 7 a.m. nor any scooter after 5 p.m.). Work out the mean number of scooters arriving during 7–8 a.m., 8–9 a.m., 9–10 a.m., and so on. Let us suppose, we get the information as illustrated in Table 4.3.

Table 4.3 *Mean Number of Scooters Arriving Per Hour*

7–8 a.m.	5.6
8–9 a.m.	5.4
9–10 a.m.	3.4
10–11 a.m.	3.6
11–12 noon	2.0
12–1 p.m.	3.0
1–2 p.m.	4.0
2–3 p.m.	6.0
3–4 p.m.	3.0
4–5 p.m.	4.0

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- Obtain the deviation of actual arrivals during a particular hour from the corresponding mean and do it for all the hours from 7 a.m. to 5 p.m. There will thus be 10 sets of 5 deviations (because of the observation on 5 days) from each of the 10 mean hourly arrivals and then prepare a frequency distribution of such deviations. Suppose we get the frequency distribution of such deviations as illustrated in Table 4.4.

Table 4.4 *Random Number Allotment*

Deviation from Mean	***Frequency***	***Percentage Frequency***	***Probability***	***Random Nos. Allotted***
+4.0	2	4	0.04	00–03
+3.5	3	6	0.06	04–09
+3.0	5	10	0.10	10–19
+2.5	4	8	0.08	20–27
+2.0	6	12	0.12	28–39
+1.5	2	4	0.04	40–43
+1.0	3	6	0.06	44–49
+0.5	4	8	0.08	50–57
–0.5	3	6	0.06	58–63
–1.0	2	4	0.04	64–67
–1.5	4	8	0.08	68–75
–2.0	2	4	0.04	76–79
–2.5	3	6	0.06	80–85
–3.0	1	2	0.02	86–87
–3.5	2	4	0.04	88–91
–4.0	4	8	0.08	92–99
Total	50	100	1.00	

In the last column of the Table 4.4 we have allotted 100 random numbers from 00 to 99, both inclusive according to the percentage frequency of the deviation or the probability distribution of deviations. Thus deviation to the extent of +4.0 from the mean, having a frequency of 2 out of 50 (and as such 4 out of 100) or a probability of 0.04, has been alloted 4 random numbers, 00 to 03. The next deviation of +3.5 with a probability of 0.06 has the next 6 random numbers, 04 to 09, allotted to it. The same treatment has been done to all the remaining deviations. The last deviation – 4.0 with a probability of 0.08 has been allotted to the last 8 random numbers, from 92 to 99.

The object of doing all this is to derive by simulation of the actual number of scooters that may be expected to arrive during any given hour. Suppose we want to know the expected number of scooters arriving on a particular day during the hour 8 to 9 a.m. The table giving the mean number of scooters arrival shows that the mean arrival during this hour is 4.4. If we can ascertain the deviation of the actual arrivals from the mean we can easily work out the actual number of scooters arrived. To do so, we look at the table of random numbers and select any two-digit number at random. Suppose, the random number so selected is 84 corresponding to which the value of the deviation of actual arrivals from the mean as per the above table is –2.5. In other words the actual arrivals for the hour 8-9 a.m. will be (4.4) – (2.5) = 2.9 (or approximately 3), so that we may say that three scooters will arrive between 8-9 a.m. on the day in question. The underlying rationale of this simulation procedure is that every deviation from its corresponding mean has the same chance of occurring by random number selection as in the actual case because each deviation has as many random numbers allotted to it as its frequency percentage in the general pool of all deviations as stated above.

We can go a step further and using the random number technique can even simulate and tell the actual arrival time of the scooter coming to the service station during any particular hour. If we are satisfied to note the arrival time correct to within, say, 5 minutes the required numbers of minutes past the hour can take a value only in one of the 12 intervals (0–5, 6–10, 11–15, ..., 56 to 60 minutes) into which any hour can be divided. Keeping this in view and the actual observations for all five days under consideration we can prepare a frequency table showing how many scooters arrive within 5 minutes, how many within 6-10 minutes past the hour and so on for the remaining intervals into which we choose to split the hour. Let the observed information for 5 days period on this basis be put as shown in Table 4.5.

Random numbers in the last column of the Table 4.5 have been allotted in a similar manner as we did in an earlier table. We have already seen that three scooters arrive during the hour 8–9 a.m. and now we want to know the actual time of their coming to service station.

Table 4.5 *Number of Scooters Arriving Within the Number of Minutes Past the Hour*

Deviation from Mean	***Frequency***	***Percentage Frequency***	***Probability***	***Random Nos. Allotted***
0–5 mts.	20	10	0.10	00–09
6–l0 mts.	30	15	0.15	10–24
11–15 mts.	10	5	0.05	25–29
16–20 mts.	40	20	0.20	30–49
21–25 mts.	16	8	0.08	50–57
26–30 mts.	14	7	0.07	58–64
31–35 mts.	18	9	0.09	65–73
36–40 mts.	12	6	0.06	74–79
41–45 mts.	16	8	0.08	80–87
46–50 mts.	14	7	0.07	88–94
51–55 mts.	6	3	0.03	95–97
56–60 mts.	4	2	0.02	98–99
Total	200	100	1.00	

For this purpose we pick a two-digit random number from the table of random numbers and let us say it is 25. A reference to the above table shows that this number occurs in the range of 25–29 which belongs to the interval 11–15 minutes and this means that the first of the 3 scooters arriving between 8–9 a.m. arrives at 15 minutes past 8 a.m. Similarly picking two more random numbers, viz., 36 and 96 we find from the above table that they are related to intervals 16–20 minutes and 51–55 minutes respectively. Thus the second scooter arrives at 20 minutes past 8 a.m. and the third scooter arrives at 55 minutes past 8 a.m.

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Proceeding in a similar manner we can also make a frequency table showing the number of scooters serviced within intervals of varying magnitudes. Each of these intervals can then be allotted a set of 100 random numbers (i.e., 00 to 99) according to the probability distribution to provide a basis for simulating the pattern of available service.

The mentioned is an example of how arrival as well as service pattern in a queuing process may be derived by Monte Carlo simulation. Remember that there is no regularity either in the arrival of the scooters or in rendering service and because of this there may be times when scooters have to wait for service while at other times the service attendant may remain idle. If in such a case we want to add one more service point to the service station then certainly we would first like to assess whether the same would be economical or not. Simulation technique can assist us in the matter. How does simulation help can better be made clear by the following example:

Example 4.8: A firm has a single channel service station with following empirical data available to its management:

(*i*) The mean arrival rate is 6.2 minutes.

(*ii*) The mean service time is 4.5 minutes.

(*iii*) The arrival and service time probability distributions are as follows:

Arrivals (Minutes)	***Probability***	***Service Time (Minutes)***	***Probability***
3-4	0.05	3-4	0.10
4-5	0.20	4-5	0.20
5-6	0.35	5-6	0.40
6-7	0.25	6-7	0.20
7-8	0.10	7-8	0.10
8-9	0.05	8-9	0.00
	1.00		1.00

The queuing process begins at 10.00 a.m. and proceeds for nearly 2 hours. An arrival goes to the service facility immediately if it is empty otherwise it will wait in a queue. The queue discipline is, First-Come First-Served or FCFS.

If the attendant's wage is ₹ 8 per hour and the customer's waiting time cost ₹ 9 per hour, would it be an economical proposition to engage second attendant? Answer on the basis of Monte Carlo simulation technique. You may use the figures based upon the simulated period for 2 hours.

Solution: From the given probability distributions of arrivals and service times, first of all we allot the random numbers to the various intervals. This has been done as follows:

Arrivals Nos. Allotted (Minutes)	*Probability*	*Random (Minutes)*	*Service*	*Probability Nos. Allotted*	*Random Time*
3–4	0.05	00–04	3–4	0.10	00–09
4–5	0.20	05–24	4.5	0.20	10–29
5–6	0.35	25–59	5–6	0.40	30–69
6–7	0.25	60–84	6–7	0.20	70–89
7–8	0.10	85–94	7–8	0.10	90–99
8–9	0.05	95–99	8–9	0.00	

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These allotted random numbers now become the basis for generating arrival and service times in conjunction with a table of random numbers. A simulation worksheet as shown on the next page has been developed in the following manner:

As the given mean arrival rate is 6.2 minutes which means that in an hour approximately 10 units arrive and as such in a simulation period for 2 hours about 20 units are expected to arrive. Hence, the number of arrivals for our simulation exercise is 20.

A table of random numbers (given in Table 4.5) is used for developing the simulation worksheet. The first random number for arrival time is 44. This number lies between 25–59 and this indicates a simulated arrival time of 5 minutes. All simulated arrival and service times have been worked out in a similar fashion.

The next step is to list the arrival time in the appropriate column. The first arrival comes in 5 minutes after the starting time. It means the attendant waited for 5 minutes. This has been shown under the column 'Waiting Time: Attendant'. The simulated service time for the first arrival is 5 minutes which results in the service being completed by 10.10 a.m. The next arrival comes at 10.11 a.m. which indicates that no one has waited in queue but the attendant has waited for 1 minute from 10.10 a.m. to 10.11 a.m. The service time ends at 10.18 a.m. However, the third arrival comes at 10.17 a.m. and the service of the second continues upto 10.18 a.m., hence the third arrival has to wait in the queue. This is shown in the column 'Waiting Time: Customer' of the simulation worksheet. One customer waiting in queue is shown in the column—'Length of the Line.' The same procedure has been followed throughout the preparation of the simulation worksheet.

The following information can be derived from the above stated simulation worksheet.

1. Average length of queue:

$$= \frac{\text{No. of customers in line}}{\text{No. of arrivals}} = \frac{13}{20} = 0.65$$

2. Average waiting time of customer before service:

$$= \frac{\text{Customer waiting time}}{\text{No. of arrivals}} = \frac{41}{20} = 2.05 \text{ minutes}$$

3. Average service time:

$$= \frac{\text{Total service time}}{\text{No. of arrivals}} = \frac{107}{20} = 5.35 \text{ minutes}$$

4. Time a customer spends in the system:

= Average service time + Average waiting time before service

= 5.35 + 2.05 = 7.40 minutes.

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Simulation worksheet developed above also states that if one more attendant is added then there is no need for a customer to 'Wait in Queue'. But the cost of having one more attendant in addition to the existing one is to be compared with the cost of one attendant and the customer waiting time. This can be worked out as under:

Two Hour Period	*Cost with*	
	One Attendant	*Two Attendants*
Customer waiting time (41 mts. × ₹ 9 per hour)	₹ 6.15	nil
Attendant's cost (2 hours × ₹ 8 per hour)	₹ 16	₹ 32
Total	₹ 22.15	₹ 32

If the above analysis is based on simulation for a period of 2 hours and is representative of the actual situation, then it can be concluded that the cost with one attendant is lower than with two attendants. Hence, it would not be an economical proposition to engage additional attendant.

Monte Carlo Simulation in the Context of Inventory Control

The following example explains the concept of Monte Carlo simulation in the content of inventory control.

Example 4.9: Suppose that the weekly demand of Electric Motors has the following probability distribution:

Number ***Numbers Assigned***	***Probability Demanded***	***Random***
0	0.10	00 to 09
1	0.40	10 to 49
2	0.30	50 to 79
3	0.20	80 to 99
Total	1.00	

Distribution pattern of delivery time was as follows:

Number of Weeks ***Assigned***	***Probability from Order to***	***Random Numbers*** ***Delivery***
2	0.20	00 to 19
3	0.60	20 to 79
4	0.20	80 to 99
Total	1.00	

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Inventory carrying cost is ₹ 5 per unit per week, order placing cost is ₹ 10 per occurrence and loss in net revenue (sale price less cost of goods) is ₹ 50 per unit from shortage.

Estimate average weekly cost of the inventory system with a policy of using reorder quantities of 4 and a reorder points of 5 units using the technique of Monte Carlo simulation for 20 weeks period taking 8 units as the opening balance of inventory.

Solution: We shall first develop the simulation worksheet, keeping all the given information in view, shown follows:

Simulation worksheet for a period of 20 weeks concerning the inventory system and the related costs with reorder point of 5 units and reorder quantity of 4 units.

Simulated Week Number	*Random Numbers (using Random Number Tables)*		*Simulated Weekly Shortage*			*Simulated Costs Activity*		
	For Demand	*For Delivery Time*	*Demand Units*	*Receipt Units*	*Balance Units*	*Inven-tory*	*Order-ing*	*Short-age*
			(in ₹)	*(in ₹)*	*(in ₹)*			
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
0					8*			
1	44		1		7	35		
2	84	82	3		4	20	10	
3	50		2		2	10		
4	85		3		(–)1			50
5	40		1		(–)1			50
6	96	88	3	4	1	5	10	
7	16		1					
8	16		1		(–1)1			50
9	97		3		(–1)3			150
10	92	39	3	4	1	5	10	
11	33		1					
12	83		3		(–3)			150
13	42	16	1	4	3	15	10	
14	07		0		3	15		
15	77	66	2	4	5	25	10	
16	50		2		3	15		
17	20		1		2	10		
18	50	95	2	4	4	20	10	
19	58		2		2	10		
20	44		1		1	5		
Total			36	20	39	190	60	450
Average			1.8	1	1.95	9.5	3	22.5

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The procedure adopted in developing the above work sheet is briefly explained as under for the guidance of the readers.

Successive Week Numbers	***Occurrence of Events and Their Effects***
1	Random number '44' indicates in column of given demand pattern that customers demanded 1 unit leaving an inventory balance of 8 – 1 = 7 units at a carrying cost of ₹ 5 per unit which equals ₹ 35.
2	Random number '84' indicates demand for 3 units and leaves an inventory balance of 4 units which is less than the reorder point of 5 units. It costs ₹ 10 to reorder. New order was placed. Random number '82' indicates that it will take 4 weeks to receive the new units ordered.
3	No special event.
4	There was demand for 3 units but only 2 units were sold because of inventory exhaustion. Under the assumption that customers went elsewhere, one unit not sold represents a loss in net revenue to the extent of ₹ 50.
5	New stock still has not arrived and a further loss of potential sales of 1 unit at a cost of ₹ 50 occurred.
6	The quantity of 4 units ordered in week, two has arrived of which 3 units were sold and 1 unit was in balance. Hence, new order was placed.
7	Neither there was any shortage nor there was anything left in inventory balance.
8 and 9	Shortage to the extent of 1 unit and unit 3, respectively remained causing a potential loss of ₹ 50 and ₹ 150, respectively.
10	Ordered quantity in week six arrived, three units were sold and one remained in balance. Fresh order was placed.
11	No shortage and no balance.
12	Shortage and potential loss of ₹ 150.
13	Ordered quantity in week 10 arrived, 1 unit was sold and three remained in balance. Fresh order was placed.
14	No special event.
15	Ordered quantity in week 13 arrived, 2 units sold and 5 units remained in balance. Fresh order was placed.
16 and 17	No special event.
18	Ordered quantity in week 25 arrived, 2 units sold and 4 remained in balance. Fresh order was placed.
19 and 20	No special event.

Under the assumptions of the simulation procedure it is noted that weekly costs averaged ₹ 9.5 to carry inventory, ₹ 3 to order and ₹ 22.5 from shortages. In all it costs ₹ 35 per week approximately to maintain the inventory system with a policy of using reorder quantities of 4 and a reorder of 5 units.

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Monte Carlo Simulation in the Context of Production Line Model

Example 4.10: A certain production process produces on the average seven per cent defective items. Defective items occur randomly. Items are packaged for sale in lots of five. The production manager wants to know what percentage of the lots contains no defective items. You are required to solve the manager's problem using Monte Carlo simulation. You may as well compare your simulated results with those obtainable using the analytical methods. If there remains any difference between such results then account for the same. (Simulate at least 10 lots each of five items.)

Solution: Since on an average the process produces 7 per cent defective items, so 93 per cent of the items produced are good. For this we can have the following probability distribution:

Items	*Probability*	*Random Nos. Assigned*
Defective	0.07	00 to 06
Good	0.93	07 to 99
Total	1.00	

Using the random number table develop the Simulation Worksheet as follows:

Simulation worksheet for 16 lots taken for finding the percentage of lots with no defective items.

Lots	*Random Numbers (Five for Each Lot)*					*Simulated Items G = Good; D = Defective*					*Defective Number in Lot*
1	44	84	82	50	85	G	G	G	G	G	0
2	40	96	88	16	16	G	G	G	G	G	0
3	97	92	39	33	83	G	G	G	G	G	0
4	42	16	07	77	66	G	G	G	G	G	0
5	50	20	50	95	58	G	G	G	G	G	0
6	44	77	11	08	38	G	G	G	G	G	0
7	87	45	09	99	81	G	G	G	G	G	0
8	97	30	36	75	72	G	G	G	G	G	0
9	79	83	07	00	42	G	G	G	D	G	1
10	13	97	16	83	11	G	G	G	G	G	0
11	45	65	34	89	12	G	G	G	G	G	0
12	64	86	46	32	76	G	G	G	G	G	0
13	07	51	25	36	19	G	G	G	G	G	0
14	32	14	31	96	03	G	G	G	G	D	1
15	93	16	62	24	08	G	G	G	G	G	0
16	38	88	74	47	00	G	G	G	G	D	1

On the basis of these simulated sample of 16 lots we can estimate the percentage of good lots as $= \frac{13}{16} \times 100 = 81.25$

If we use the analytical method or well formulated mathematical method, we get the following result:

$$p = p.\text{ (Good Item)} = 0.93$$
$$q = p.\text{ (Defective Item)} = 0.07$$

Each lot is of 5 items which means that $n = 5$.

NOTES

Using the Binomial probability function we have the following:

$$p\text{ (All five good items)} = {}^5C_5\,(0.93)^5\,.(0.07)^0$$
$$= 1.(0.93)^5.1$$
$$= 0.70$$
$$= 70\%$$

This analytical answer equals the true expected value or the average number of good lots over the long run. However, the simulated result gives this percentage as 81.25 which differs from that of 70 per cent obtained by mathematical method. The difference is on account of the fact that we took only 16 lots for simulation exercise. If we increase this number, then our answer will approximate to that of the answer from analytical method. Thus, simulation gives only the best possible estimates and not the optimal result as given by analytical methods.

4.8 SIMULATION AND RANDOM VARIABLE: MATHEMATICAL APPROACH FOR RANDOMNESS

Simulation modelling is the best thing to observe a real system. It differs from mathematical modelling in that the relationship between the input and output need not be stated explicitly. Instead, it breaks down the real system into small modules and then imitates the actual behaviour of the system by using logical relationships to link the modules together. Starting with the input module, the simulation computations move among the appropriate modules until the output result is realized.

Simulation computations, though usually simple and luminous, are voluminous. It is thus unthinkable to execute a simulation model without the use of the computer.

Simulation models are much more flexible in representing systems than their mathematical counterparts. The main reason for this flexibility is that simulation views the system at elemental level, whereas mathematical models tend to represent the system from a more global standpoint.

The flexibility of simulation is not without drawbacks. The development of a simulation model is usually costly in both time and resources. Moreover, the execution of some simulation models, even on the fastest computers, may be slow. Besides this, quantification of some of the crucial variables that affect the system may not be possible, though introduction of dummy or proxy variables can serve as surrogates to some extent.

Simulation, akin to the incremental principle, does not produce optimum results. It can at best say whether a particular decision policy (a course of action) out of a set of various alternative policies is better than the rest or not. Thus, unless the list of all possible courses of action is exhaustive, simulation, on its own cannot dedue out optimal values. However, as simulation offers the solution by allowing experimentation with a model of the system without interfering with the real system, it is frequently used as a bypass for complex mathematical analysis in view of its practical utility.

Monte Carlo technique has been used to tackle a variety of problems involving stochastic situations and mathematical problems, which cannot be solved with mathematical techniques and where physical experimentation with the actual system is impracticable. The stochastic situations are usually a long sequence of probabilistic events or steps. We may be able to write mathematical formulae for probability of a particular event, but to write a mathematical relationship for the probabilities of all events in the sequence is a difficult task.

In contrast to mathematical modelling where the results of the analysis yield a direct and overal solution to the problem, in simulation, the behaviour of the system is observed over a sufficiently long period of time, and in the process, the relevant information is collected. The system is first described by listing the various events in the order of their occurrence. An event representing a point in time signifies the end of one or more activities and the beginning of the next activity. As each event occurs, certain actions are taken, resulting in the generation of new events which are further considered in sequence. For example, the arrival of a customer at a service facility is the occurrence of an event and action taken depends upon the availability of facility which may be free or occupied. If free, service begins, if occupied, wait in line, which further generates new activities. If service begins, compute service time and if to wait in queue, compute waiting time and length of queue, etc. The process is repeated for the next arrival.

The experimentation is performed on a simulated model of the real system. It is a sort of sampling technique in which, instead of drawing samples from a real population, the samples are drawn from a theoretical equivalent of the real population. By making use of roulette wheel or random numbers, Monte Carlo approach determines the probability distribution of the occurrence of the event under consideration and then samples the data from this distribution.

Important applications of Monte Carlo method are found in waiting line problems, formulating maintenance policies, determining the inventory level, etc. Though the process of general simulation can be carried on physical (or iconic), analog (or schematic) and mathematical (or symbolic) models, in the subject of mathematics or any of its offshoots like operations research, it is only the mathematical model that is what is implied.

Steps Involved In Monte Carlo Simulation

1. Establish a cumulative distribution function.
2. Set up the table and assign tag numbers with the help of cumulative distribution function. The tag numbers are assigned in such a way as to reflect the probability of the various events.
3. Obtain the random numbers from a random number table. While using a random number table, one can start at any point in the table and proceed in any direction, but one should not choose or select the random numbers.

Notes:

1. A random number is a number in a sequence of numbers whose probability of occurrence is the same as that of any other number in the sequence.
2. Random numbers can be generated by any one of the following four methods:
   (*i*) Manual generation (lottery method, roulette wheel, etc.).
   (*ii*) Arithmetic method (mid-square method, mixed/additive/multiplicative congruential methods).
   (*iii*) Computer methods.
   (*iv*) Using random number tables.

Example 4.11: A confectioner sells confectionary items. Post data of demand per week (in hundred kilogramms) with frequency of occurrence is given below:

Demand/Week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using the following sequence of random numbers, generate the demand for the next 10 weeks. Also find the average demand per week.

35, 52, 90, 13, 23, 73, 34, 57, 35, 83.

Solution: The tag numbers (random number intervals) are found as follows:

Demand	*Probability (or % Frequency)*	*Cumulative Probability*	*Tag Number (random Number Interval)*
0	0.04	0.04	00–03
5	0.22	0.26	04–25
10	0.16	0.42	26–41
15	0.42	0.84	42–83
20	0.10	0.94	84–93
25	0.06	1.00	94–99

Determination of Demand Levels

Trial No. (Week No.)	*Random Number*	*Range*	*Demand*
1	35	26–41	10
2	52	42–83	15
3	90	84–93	20
4	13	04–25	5
5	23	04–25	5
6	73	42–83	15
7	34	26–41	10
8	57	42–83	15
9	35	26–41	10
10	83	42–83	15

Average demand/week:

$$= \frac{(10+15+20+5+5+15+10+15+10+15)}{10}$$

$$= \frac{120}{10} = 12 \text{ hundred kg} = 1200 \text{ kg}$$

NOTES

Expected demand/week:

$= 0.04(0) + 0.22(5) + 0.16(10) + 0.42(15) + 0.10(20) + 0.06(25)$

$= 14.5$ hundred kg

$= 1450$ kg

Remark

As the number of trials increases, the average demand comes closer to the expected demand. Thus, by increasing the number of trials, a much more accurate value can be obtained. Sometimes, when a variable is continuous, with numerical values significant to third or fourth decimal, for the sake of accuracy, we have to go in for three digited or four digited random numbers accordingly while simultaneously increasing the number of trials for arriving at much more accurate results.

Example 4.12: Generate a sequence of 5 two-digited random numbers by employing the following methods:

(*i*) Mixed Congruential Method

(*ii*) Multiplicative Congruential Method

(*iii*) Additive Congruential Method

The recursive equation for the mixed congruential method is $r_{i+1} = (21r_i + 53)$ (modulo 100) and seed, $r_0 = 46$.

(*iv*) Midsquare Method (Using the same seed)

Solution: (*i*) Using mixed congruential method:

Recursive Relation $= r_{i+1} = (21r_i + 53)$ *(modulo 100)*

Recursive Relation	***Dividend***	***Divisor***	***Remainder (Residue)***
$r_1 = (21r_0 + 53)$ (modulo 100)	$21r_0+53$ $= 21(46) + 53$ $= 1019$	100	$r_1 = 19$
$r_2 = (21r_1 + 53)$ (modulo 100)	$(21r_1+ 53)$ $= 21(19) + 53$ $= 452$	100	$r_2 = 52$
$r_3 = (21r_2 + 53)$ (modulo 100)	$(21r_2 + 53)$ $= 21(52) + 53$ $= 1145$	100	$r_3 = 45$
$r_4 = (21r_3 + 53)$ (modulo 100)	$(21r_3 + 53)$ $= 21(45) + 53$ $= 978$	100	$r_4 = 78$
$r_5 = (21r_4 + 53)$ (modulo 100)	$(21r_4 + 53)$ $= 21(78) + 53$ $= 1691$	100	$r_5 = 91$

NOTES

(*ii*) Using multiplicative congruential method:

Recursive Relation = $r_{i+1} = (21r_i)$ (modulo 100)

Recursive Relation	*Dividend*	*Divisor*	*Remainder (Residue)*
$r_1 = (21r_0)$ (modulo 100)	$21r_0 = 21 \times 46$ $= 966$	100	$r_1 = 66$
$r_2 = (21r_1)$ (modulo 100)	$21r_1 = 21 \times 66$ $= 1386$	100	$r_2 = 86$
$r_3 = (21r_2)$ (modulo 100)	$21r_2 = 21\times86$ $= 1806$	100	$r_3 = 06$
$r_4 = (21r_3)$ (modulo 100)	$21r_3 = 21 \times 6$ $= 126$	100	$r_4 = 26$
$r_5 = (21r_4)$ (modulo 100)	$21r_4 = 21 \times 26$ $= 546$	100	$r_5 = 46$

(*iii*) Using additive congruential method:

Recursive Relation: $r_{i+1} = (r_i + 53)$ (modulo 100)

Recursive Relation	*Dividend*	*Divisor*	*Remainder (Residue)*
$r_1 = (r_0 + 53)$ (modulo 100)	$r_0 + 53$ $= 46 + 53 = 99$	100	$r_1 = 99$
$r_2 = (r_1 + 53)$ (modulo 100)	$r_1 + 53$ $= 99 + 53 = 152$	100	$r_2 = 52$
$r_3 = (r_2 + 53)$ (modulo 100)	$r_2 + 53$ $= 52 + 53 = 105$	100	$r_3 = 05$
$r_4 = (r_3 + 53)$ (modulo 100)	$r_3 + 53$ $= 5 + 53 = 58$	100	$r_4 = 58$
$r_5 = (r_4 + 53)$ (modulo 100)	$r_4 + 53$ $= 58 + 53 = 111$	100	$r_5 = 11$

The remainders so arrived at constitute the respective random numbers.

(*iv*) Using midsquare method:

$r_0 = 46$; *Recursive Relation:*

r_{i+1} = *Number formed by the middle two digits of* r_2

$46^2 = 2116$ $\quad r_1 = 11$

$11^2 = 0121$ $\quad r_2 = 12$

$12^2 = 0144$ $\quad r_3 = 14$

$14^2 = 0256$ $\quad r_4 = 25$

$25^2 = 0625$ $\quad r_5 = 62$

Note: Generally, the random numbers generated by midsquare method results in repeated sequence of random numbers (i.e., they form small loops which are not sufficient for generating random numbers).

Remark

The random numbers generated in all the preceding four methods are not truly random (and are called pseudorandom numbers). However, these numbers (corresponding to mixed congruential method in particular) are treated as random as they pass a certain number of statistical tests for randomness. In fact, tables of random numbers are available and these are generated using some random physical process (electric current in Rand case) and are considered to be truly random numbers.

NOTES

Case Study 1: Economical Crew Option

Maintenance of machines in a factory can be carried out by one-man or sometimes by a two-man crew. The times taken with one-man crew are 1.5, 2.0, 2.5 or 3.0 hours with probabilities of 0.20, 0.30, 0.35 and 0.15, respectively. A two-man crew requires 0.75, 1.00, 1.50 or 2.00 hours with probabilities of 0.25, 0.35, 0.20, and 0.20 respectively. The cost of labour is ₹ 7 per hour and the overhead expenses are ₹ 3 per hour per crew. Decide whether one or two-man crew is more economical.

Solution: First fix tag numbers for two types of crews as follows:

One-Man Crew

Time (Hrs.)	*Probability*	*Cumulative Probability*	*Tag Number (Random Number Interval)*
1.5	0.20	0.20	00–19
2.0	0.30	0.50	20–49
2.5	0.35	0.85	50–84
3.0	0.15	1.00	85–99

Two-Man Crew

Time (Hrs.)	*Probability*	*Cumulative Probability*	*Tag Number (Random Number Interval)*
0.75	0.25	0.25	00–25
1.00	0.35	0.60	25–59
1.50	0.20	0.80	60–79
2.00	0.20	1.00	80–99

One-Man Crew			**Two-Man Crew**		
Trial Number	*Random Number*	*Time Taken (Hrs.)*	*Trial Number*	*Random Number*	*Time Taken (Hrs.)*
1	39	2.0	1	29	1.00
2	00	1.5	2	82	2.00
3	35	2.0	3	08	0.75
4	04	1.5	4	43	1.00
5	12	1.5	5	17	0.75
6	11	1.5	6	19	0.75
7	23	2.0	7	40	1.00
8	18	1.5	8	62	1.50
9	83	2.5	9	49	1.00
10	35	2.0	10	27	1.00
11	50	2.5	11	50	1.00
12	52	2.5	12	77	1.50
13	68	2.5	13	71	1.50
14	29	2.0	14	60	1.50
15	23	2.0	15	47	1.00

Average $= \frac{29.5}{15} = 1.97$ hours Average $= \frac{17.25}{15} = 1.15$ hours

Cost with one-man crew = ₹ $(1.97 \times 7) + (1.97 \times 3)$

= ₹ 19.70

Cost with two-man crew = ₹ $(1.15 \times 2 \times 7) + (1.15 \times 3)$ = ₹ 19.55

From the above, it is evident that two-man crew is more economical.

NOTES

Example 4.13: Use simulation to approximate the value of $\int_0^1 e^{-x}\,dx$.

Solution: Let $y = e^{-x}$

When $x = 0, \quad y = e^{-0}$

$= e^0 = 1$

When $x = 1, \quad y = e^{-1}$

$= \frac{1}{e} = 0.368$

The given definite integral stands for the shaded are in the adjoining graph.

The random numbers are so chosen that $0 < R_1 < 1$ and $0 < R_2 < 1$.

Trial	$R_1 = x$	$R_2 = y$	e^{-x}	*Hit (H) or Miss (M)*
1	0.48	0.22	0.62	H
2	0.51	0.62	0.61	M
3	0.22	0.31	0.80	H
4	0.22	0.31	0.80	H
5	0.80	0.23	0.45	H
6	0.56	0.07	0.57	H
7	0.06	0.93	0.94	H
8	0.92	0.44	0.40	M
9	0.51	0.20	0.61	H
10	0.13	0.26	0.88	H
11	0.65	0.93	0.52	M
12	0.60	0.01	0.55	H
13	0.51	0.17	0.60	H
14	0.50	0.49	0.61	H
15	0.13	0.58	0.88	H

Note:

Hit (H), if $y < e^{-x}$ and Miss (M) if $y > e^{-x}$

No. of Trials = 15

No. of Hits = 12

Ratio of Hits to Trials = $\frac{12}{15} = 0.8$

Area of the Square ABCD = 1 × 1 = 1 unit square

$\therefore$ Shaded area = Value of the definite integral = $\int_0^1 e^{-x}\,dx$

$= 0.8 \times 1 = 0.8$ sq. units

If the no. of trials and the no. of digits of random numbers are increased, the accuracy in the result will increase. The true value of the integral is:

$$I = \int_0^1 e^{-x}\,dx = \left[\frac{e^{-x}}{-1}\right]_0^1$$

$$= -\left[e^{-x}\right]_0^1 = -\left[e^{-1} - e^{-0}\right]$$

$$= -\left(\frac{1}{e}-1\right) = 1 - \frac{1}{e} = 0.632$$

NOTES

Example 4.14: Use Monte Carlo simulation to approximate the value of $\int_3^5 x^2 dx$.

Solution: Let $y = x^2$

When, $x = 0,$

$y = 0^2 = 0$

When, $x = 3,$

$y = 3^2 = 9$

When, $x = 5,$

$y = 5^2 = 25$

The given definite integral stands for the shaded area in the adjoining graph.

The random numbers are so chosen that $0 < R_1 < 1$ and $0 < R_2 < 1$.

Trial No.	*R_1 (Random No. for x Co-Ordinate)*	*R_2 (Random No. for y Co-Ordinate)*	*$x = 3 + 2R_1$*	*$y = 25R_2$*	*x_2*	*Hit (H) or Miss (M) (H, if $y < x_2$ M, if $y > x_2$)*
1	0.21	0.96	3.42	24.00	11.696	M
2	0.11	0.82	3.22	20.50	10.368	M
3	0.71	0.56	4.42	14.00	19.536	H
4	0.65	0.68	4.30	17.00	18.490	H
5	0.41	0.58	3.82	14.50	14.592	H
6	0.35	0.40	3.70	10.00	13.690	H
7	0.17	0.82	3.34	20.50	11.156	M
8	0.91	0.11	4.82	2.75	23.232	H
9	0.07	0.34	3.14	8.50	9.860	H
10	0.34	0.25	3.68	6.25	13.542	H

No. of Trials = 10

No. of Hits = 7

Ratio of hits to Trials = $\frac{7}{10}$

Area of the Rectangle ABC = 2 × 25 = 50 sq. units

$\therefore$ Shaded area = $\frac{7}{10} \times 50$

= 35 sq. units

By increasing the no. of trials and the no. of digits of the random numbers, we can arrive at much more accurate results.

The actual value of the integral:

$$\int_3^5 x^2 dx = \left[\frac{x^3}{3}\right]_3^5$$

$$= \left(\frac{5^3 - 3^3}{3}\right)$$

$$= \left(\frac{125-27}{3}\right)$$

= 32.67 sq. units

4.9 RANDOM VARIABLE AND STANDARD DISTRIBUTION FUNCTIONS

4.9.1 Random Variable

A random variable takes on different values as a result of the outcomes of a random experiment. In other words, a function which assigns numerical values to each element of the set of events that may occur (i.e., every element in the sample space) is termed as random variable. The value of a random variable is the general outcome of the random experiment. One should always make a distinction between the random variable and the values that it can take on. All these can be illustrated by a few examples shown in Table 4.6.

Table 4.6 *Random Variable*

Random Variable	*Values of the Random Variable*	*Description of the Values of the Random Variable*
X in	0, 1, 2, 3, 4	Possible number of heads four tosses of a fair coin
Y	1, 2, 3, 4, 5, 6	Possible outcomes in a single throw of a die
Z	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	Possible outcomes from throwing a pair of dice
M	0, 1, 2, 3, ... S	Possible sales of newspapers by a newspaper boy, S representing his stock

All the stated random variable assignments cover every possible outcome and each numerical value represents a unique set of outcomes. A random variable can be either discrete or continuous. If a random variable is allowed to take on only a limited number of values, it is a discrete random variable, but if it is allowed to assume any value within a given range, it is a continuous random variable. Random variables presented in Table 4.6 are examples of discrete random variables. We can have continuous random variables if they can take on any value within a range of values, for example, within 2 and 5, in that case we write the values of a random variable x as,

$$2 \le x \le 5$$

Techniques of Assigning Probabilities

We can assign probability values to the random variables. Since the assignment of probabilities is not an easy task, we should observe following rules in this context:

(*i*) A probability cannot be less than zero or greater than one, i.e., $0 \le pr \le 1$, where pr represents probability.

(*ii*) The sum of all the probabilities assigned to each value of the random variable must be exactly one.

There are three techniques of assignment of probabilities to the values of the random variable that are as follows:

(*i*) ***Subjective Probability Assignment:*** It is the technique of assigning probabilities on the basis of personal judgement. Such assignment may differ from individual to individual and depends upon the expertise of the person assigning the probabilities. It cannot be termed as a rational way of assigning probabilities, but is used when the objective methods cannot be used for one reason or the other.

NOTES

(*ii*) ***A-Priori Probability Assignment:*** It is the technique under which the probability is assigned by calculating the ratio of the number of ways in which a given outcome can occur to the total number of possible outcomes. The basic underlying assumption in using this procedure is that every possible outcome is likely to occur equally. However, at times the use of this technique gives ridiculous conclusions. For example, we have to assign probability to the event that a person of age 35 will live upto age 36. There are two possible outcomes, he lives or he dies. If the probability assigned in accordance with a-priori probability assignment is half then the same may not represent reality. In such a situation, probability can be assigned by some other techniques.

(*iii*) ***Empirical Probability Assignment:*** It is an objective method of assigning probabilities and is used by the decision-makers. Using this technique the probability is assigned by calculating the relative frequency of occurrence of a given event over an infinite number of occurrences. However, in practice only a finite (perhaps very large) number of cases are observed and relative frequency of the event is calculated. The probability assignment through this technique may as well be unrealistic, if future conditions do not happen to be a reflection of the past.

Thus, what constitutes the 'best' method of probability assignment can only be judged in the light of what seems best to depict reality. It depends upon the nature of the problem and also on the circumstances under which the problem is being studied.

4.9.1 Probability Distribution Functions: Discrete and Continuous

When a random variable x takes discrete values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$, we have a discrete probability distribution of X.

The function $p(x)$ for which $X = x_1, x_2, \ldots, x_n$ takes values $p_1, p_2, \ldots, p_n$, is the probability function of X.

The variable is discrete because it does not assume all values. Its properties are:

$$p(x_i) = \text{Probability that } X \text{ assumes the value } x$$

$$= \text{Prob } (x = x_i) = p_i$$

$$p(x) \geq 0, \ \Sigma p(x) = 1$$

NOTES

For example, four coins are tossed and the number of heads X noted. X can take values 0, 1, 2, 3, 4 heads.

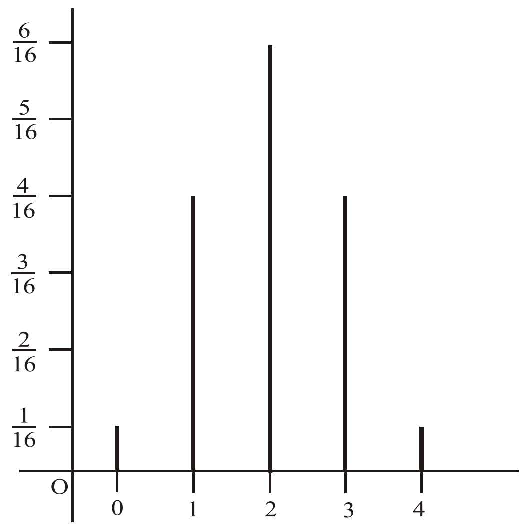
$$p(X=0) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$p(X=1) = {}^4C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 = \frac{4}{16}$$

$$p(X=2) = {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{6}{16}$$

$$p(X=3) = {}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) = \frac{4}{16}$$

$$p(X=4) = {}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0 = \frac{1}{16}$$



$$\sum_{x=2}^{4} p(x) = \frac{1}{16}+\frac{4}{16}+\frac{6}{16}+\frac{4}{16}+\frac{1}{16} = 1$$

This is a discrete probability distribution (see Example 4.3).

Example 4.15: If a discrete variable X has the following probability function, then find (*i*) a (*ii*) $p(X \le 3)$ (*iii*) $p(X \ge 3)$.

Solution: The solution is obtained as follows:

x_1	$p(x_i)$
0	0
1	a
2	$2a$
3	$2a^2$

4	$4a^2$
5	$2a$

Since $\Sigma p(x) = 1$, $0 + a + 2a + 2a^2 + 4a^2 + 2a = 1$

NOTES

$\therefore \quad 6a^2 + 5a - 1 = 0$, so that $(6a - 1)(a + 1) = 0$

$$a = \frac{1}{6} \text{ or } a = -1 \text{ (Not admissible)}$$

For $a = \frac{1}{6}$, $p(X \le 3) = 0 + a + 2a + 2a^2 = 2a^2 + 3a = \frac{5}{9}$

$$p(X \ge 3) = 4a^2 + 2a = \frac{4}{9}$$

Discrete Distributions

There are several discrete distributions. Some other discrete distributions are described as follows:

(i) Uniform or Rectangular Distribution

Each possible value of the random variable x has the same probability in the uniform distribution. If x takes vaues $x_1, x_2 ...., x_k$, then,

$$p(x_i, k) = \frac{1}{k}$$

The numbers on a die follow the uniform distribution,

$$p(x_i, 6) = \frac{1}{6} \text{ (Here, } x = 1, 2, 3, 4, 5, 6)$$

Bernoulli Trials

In a Bernoulli experiment, an Event E either happens or does not happen (E'). Examples are, getting a head on tossing a coin, getting a six on rolling a die, and so on.

The Bernoulli random variable is written,

$$X = 1 \text{ if } E \text{ occurs}$$
$$= 0 \text{ if } E' \text{ occurs}$$

Since there are two possible values it is a case of a discrete variable where,

Probability of success $= p = p(E)$

Profitability of failure $= 1 - p = q = \text{p}(E')$

We can write,

For $k = 1$, $f(k) = p$

For $k = 0$, $f(k) = q$

For $k = 0$ or 1, $f(k) = p^k q^{1-k}$

NOTES

Negative Binomial

In this distribution, the variance is larger than the mean.

Suppose, the probability of success p in a series of independent Bernoulli trials remains constant.

Suppose the rth success occurs after x failures in $x + r$ trials, then

(*i*) The probability of the success of the last trial is p.

(*ii*) The number of remaining trials is $x + r - 1$ in which there should be $r - 1$ successes. The probability of $r - 1$ successes is given by,

$$^{x+r-1}C_{r-1} p^{r-1} q^x$$

The combined pobability of Cases (*i*) and (*ii*) happening together is,

$$p(x) = px^{x+r-1}C_{r-1} p^{r-1} q^x \quad x = 0, 1, 2,....$$

This is the Negative Binomial distribution. We can write it in an alternative form,

$$p(x) = {}^{-r}C_x p^r (-q)^x \qquad x = 0, 1, 2,....$$

This can be summed up as follows:

In an infinite series of Bernoulli trials, the probability that $x + r$ trials will be required to get r successes is the negative binomial,

$$p(x) = {}^{x+r-1}C_{r-1} p^{r-1} q^x \qquad r \geq 0$$

If $r = 1$, it becomes the geometric distribution.

If $p \to 0, \to \infty, rp = m$ a constant, then the negative binomial tends to be the Poisson distribution.

(*ii*) *Geometric Distribution*

Suppose the probability of success p in a series of independent trials remains constant.

Suppose, the first success occurs after x failures, i.e., there are x failures preceding the first success. The probability of this event will be given by $p(x) = q^x p$ $(x = 0, 1, 2,.....)$

This is the geometric distribution and can be derived from the negative binomial. If we put $r = 1$ in the negative binomial distribution, then

$$p(x) = {}^{x+r-1}C_{r-1} p^{r-1} q^x$$

We get the geometric distribution,

$$p(x) = {}^xC_0 p^1 q^x = pq^x$$

$$\Sigma p(x) = \sum_{n=0}^{p} q^x p = \frac{p}{1-q} = 1$$

$$E(x) = \text{Mean} = \frac{p}{q}$$

$$\text{Variance} = \frac{p}{q^2}$$

$$\text{Mode} = \left(\frac{1}{2}\right)^x$$

NOTES

(*iii*) *Hypergeometic Distribution*

From a finite population of size N, a sample of size n is drawn without replacement.

Let there be N_1 successes out of N.

The number of failures is $N_2 = N - N_1$.

The disribution of the random variable X, which is the number of successes obtained in the discussed case, is called the hypergeometic distribution.

$$p(x) = \frac{{}^{N_1}C_x \, {}^{N}C_{n-x}}{{}^{N}C_n} \quad (X = 0, 1, 2, ..., n)$$

Here, x is the number of successes in the sample and $n-x$ is the number of failures in the sample.

It can be shown that,

$$\text{Mean} : E(X) = n\frac{N_1}{N}$$

$$\text{Variance} : \text{Var}(X) = \frac{N-n}{N-1}\left(\frac{nN_1}{N} - \frac{nN_1^2}{N}\right)$$

(*iv*) *Multinomial*

There are k possible outcomes of trials, viz., $x_1, x_2, ..., x_k$ with probabilities $p_1, p_2, ..., p_k$, n independent trials are performed. The multinomial distibution gives the probability that out of these n trials, x_1 occurs n_1 times, x_2 occurs n_2 times, and so on. This is given by $\frac{n!}{n_1!n_2!....n_k!} p_1^{n_1} p_2^{n_2} ....p_k^{n}$

Where, $\sum_{i=1}^{k} n_i = n$

Example 4.16: For the following probability distribution, find $p(x > 4)$ and $p(x \geq 4)$:

x	0	1	2	3	4	5
$p(x)$	0	a	$a/2$	$a/2$	$a/4$	$a/4$

Solution:

Since, $\Sigma p(x) = 1, 0 + a + \frac{a}{2} + \frac{a}{2} + \frac{a}{4} + \frac{a}{4} = 1$

$\therefore \quad \frac{5}{2}a = 1 \quad \text{or} \quad a = \frac{2}{5}$

$$p(x > 4) = p(x = 5) = \frac{9}{4} = \frac{1}{10}$$

$$p(x \le 4) = 0 + a + \frac{a}{2} + \frac{a}{2} + \frac{a}{4} + \frac{9a}{4} = \frac{9}{10}$$

NOTES

Example 4.17: A fair coin is tossed 400 times. Find the mean number of heads and the corresponding standard deviation.

Solution: This is a case of Binomial distribution with $p = q = \frac{1}{2}, n = 400$.

The mean number of heads is given by $\mu = np = 400 \times \frac{1}{2} = 200$.

and S. D. $\sigma = \sqrt{npq} = \sqrt{400 \times \frac{1}{2} \times \frac{1}{2}} = 10$

Example 4.18: A manager has thought of 4 planning strategies each of which has an equal chance of being successful. What is the probability that at least one of his strategies will work if he tries them in 4 situations? Here $p = \frac{1}{4}, q = \frac{3}{4}$.

Solution: The probability that none of the strategies will work is given by,

$$p(0) = {}^4C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right)^4$$

The probability that at least one will work is given by $1 - \left(\frac{3}{4}\right)^4 = \frac{175}{256}$.

Example 4.19: For the Poisson distribution, write the probabilities of 0, 1, 2, ... successes.

Solution: The solution is obtained as follows:

x	$p(x) = e^{-m}\frac{m^x}{x!}$
0	$p(0) = e^{-m} m^0 / 0!$
1	$p(1) = e^{-m}\frac{m}{1!} = p(0).m$
2	$e^{-m}\frac{m^2}{2!} = p(2) = p(1).\frac{m}{2}$
3	$e^{-m}\frac{m^3}{3!} = p(3) = p(2).\frac{m}{3}$
$\vdots$	

and so on.

Total of all probabilities $\Sigma p(x) = 1$.

Example 4.20: What are the raw moments of Poisson distribution?

Solution:

First raw moment $\mu'_1 = m$

Second raw moment $\mu'_2 = m^2 + m$

Third raw moment $\mu'_3 = m^3 + 3m^2 + m$

NOTES

(*v*) Continuous Probability Distributions

When a random variate can take any value in the given interval $a \le x \le b$, it is a continuous variate and its distribution is a continuous probability distribution.

Theoretical distributions are often continuous. They are useful in practice because they are convenient to handle mathematically. They can serve as good approximations to discrete distributions.

The range of the variate may be finite or infinite.

A continuous random variable can take all values in a given interval. A continuous probability distribution is represented by a smooth curve.

The total area under the curve for a probability distribution is necessarily unity. The curve is always above the x axis because the area under the curve for any interval represents probability and probabilities cannot be negative.

If X is a continous variable, the probability of X falling in an interval with end points z_1, z_2 may be written $p(z_1 \le X \le z_2)$.

This probability corresponds to the shaded area under the curve in Figure 4.1.

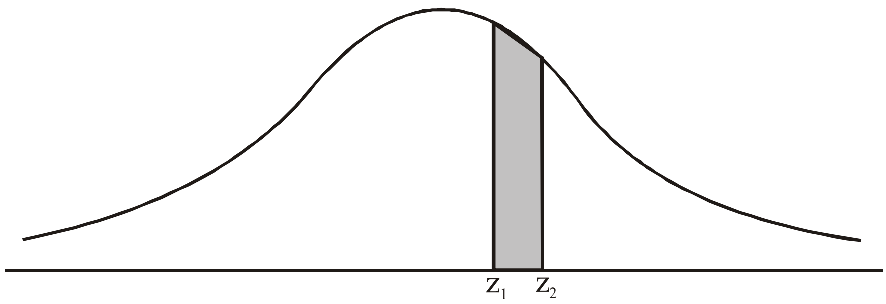


Fig. 4.1 *Continuous Probability Distribution*

A function is a probability density function if,

$\int_{-\infty}^{\infty} p(x)dx = 1, p(x) \geq 0, -\infty < x < \infty,$ i.e., the area under the curve $p(x)$ is 1 and the probability of x lying between two values a, b, i.e., $p(a < x < b)$ is positive. The most prominent example of a continuous probability function is the normal distribution.

Cumulative Probability Function (CPF)

The Cumulative Probability Function (CPF) shows the probability that x takes a value less than or equal to, say, z and corresponds to the area under the curve up to z:

$$p(x \leq z) = \int_{-\infty}^{z} p(x)dx$$

This is denoted by $F(x)$.

NOTES

4.9.2 Extension to Bivariate Case: Elementary Concepts

If in a bivariate distribution the data is quite large, then they may be summed up in the form of a two-way table. In this for each variable, the values are grouped into different classes (not necessary same for both the variables), keeping in view the same considerations as in the case of univariate distribution. In other words, a bivariate frequency distribution presents in a table pairs of values of two variables and their frequencies.

For example, if there is m classes for the $X-$ variable series and n classes for the $Y-$ variable series then there will be $m \times n$ cells in the two-way table. By going through the different pairs of the values (x, y) and using tally marks, we can find the frequency for each cell and thus get the so called bivariate frequency table as shown in Table 4.7.

Table 4.7 *Bivariate Frequency Table*

x Series → / Y Series ↓	Classes / Mid Points: x_1 x_1 ....x .... x_m	Total of Frequencies of y
Classes / Mid Points: y_1, y_2, y, ..., y_n	$f(x, y)$	f_y
Total of Frequencies of x	f_x	Total $f_x =$ $f_y =$ N

Here, $f(x,y)$ is the frequency of the pair (x, y). The formula for computing the correlation coefficient between x and y for the bivariate frequency table is,

$$r = \frac{N\Sigma xyf(x, y) - (\Sigma xfx)(\Sigma yfy)}{\sqrt{\left[N\Sigma x^2 f_x - (\Sigma xfx)^2\right] \times \left[N\Sigma y^2 f_y - (\Sigma yfy)^2\right]}}$$

Where, N is the total frequency.

Check Your Progress

7. What is the main purpose of simulation in management?
8. What are Monte Carlo methods?
9. Write one difference between analytical solution and Monte Carlo simulation.
10. What is Bayes' theorem?
11. What is joint probability?
12. When a distribution is said to be symmetrical?
13. What is continuous probability distribution?

NOTES

4.10 ANSWERS TO 'CHECK YOUR PROGRESS'

1. Random variables are classified according to their probability density function.
2. Random numbers are called pseudorandom numbers when they are generated by some deterministic process, but have already qualified the pre-determined statistical test for randomness.
3. Monte Carlo simulation needs the generation of a sequence of random numbers, which constitute an integral part of the simulation model and also help in determining random observations from the probability distribution.
4. Simulation technique is specifically used to imitate the function of a real world system that develops over time.
5. A system is a collection of entities that act and interact toward the accomplishment of some logical end. Generally, the systems are referred to be dynamic as their status changes over time.
6. A Linear Congruential Generator (LCG) represents one of the oldest and well known pseudorandom number generator algorithms.
7. The main purpose of simulation in management is to provide feedback, which is vital for the learning process.
8. Monte Carlo methods are basically the algorithms used in the computation of result to be calculated from repeated random sampling. These methods help in computerized calculations because these can perform repeated computation using random or pseudorandom numbers.
9. Analytical solution produces the optimal answer to a given problem, while Monte Carlo simulation yields a solution which should be very close to the optimal but not necessarily the exact correct solution.
10. Bayes' theorem makes use of conditional probability formula where the condition can be described in terms of the additional information which would result in the revised probability of the outcome of an event.
11. The product of prior probability and conditional probability for each state of nature is called joint probability.
12. If $p = 0.5$, the distribution is symmetrical.

13. When a random variate can take any value in the given interval $a \leq x \leq b$, it is a continuous variate and its distribution is a continuous probability distribution.

NOTES

4.11 SUMMARY

- Simulation is the most widely used flexible modelling approach. This approach is used to model the behaviour of individual components within the system with the help of random sampling technique for generating pragmatic variability.
- To determine the behaviour of a real system in actual environment, a number of experiments are performed on simulated models either in the laboratories or on the computer itself.
- The random variable is a real-valued function, defined over a sample space associated with the outcome of a conceptual chance experiment. Random variables are classified according to their probability density function.
- Random number refers to a uniform random variable or a numerical value assigned to a random variable, following uniform probability density function. In other words, it is a number in a sequence of numbers, whose probability of occurrence is the same as that of any other number in that sequence.
- Random numbers are called pseudorandom numbers when they are generated by some deterministic process, but have already qualified the pre-determined statistical test for randomness.
- The Monte Carlo method is a simulation technique in which statistical distribution functions are created by using a series of random numbers. The method is generally used to solve the problems that cannot be adequately represented by the mathematical models or where the solution of the model cannot be arrived at, by analytical method.
- Monte Carlo simulation needs the generation of a sequence of random numbers, which constitute an integral part of the simulation model and also help in determining random observations from the probability distribution.
- Simulation is the process of designing a mathematical or logical model of a real system and then accomplishing experiments with the model to describe, explain and predict the behavior of the real system.
- Simulation is very useful in understanding the expected performance of the real system and also in testing the efficiency of the system design. A static simulation model represents a system at a particular point in time whereas a dynamic simulation model represents a system developed over time.
- An event is characteristically defined as a situation that causes the state of the system to change instantaneously. When the state of system transforms/changes only at discrete points in time termed as 'discrete events'. When the state of system changes continuously over time, then it is termed as 'continuous event'.
- A Linear Congruential Generator (LCG) represents one of the oldest and well known pseudorandom number generator algorithms.

- By using a fresh series of random numbers at the appropriate junctures we can also examine the reactions of the simulated model just as if the same alterations had actually been made in the system itself. Monte Carlo simulation, therefore, provides a tool of knowing in advance whether or not the expense to be incurred or the investment to be made in making the changes envisaged.
- The main purpose of simulation in management is to provide feedback, which is vital for the learning process. It creates an atmosphere in which managers play a dynamic role by enriching their experience through involvement in reckoning with actual conditions through experimentation on paper.
- Various simulation models, based on the principle of similitude (such as model of aeroplanes initiating flight conditions in a wind tunnel) have been in use for a long time. However, Monte Carlo simulation is a recent Operations Research or OR innovation.
- Simulation computations, though usually simple and luminous, are voluminous. It is thus unthinkable to execute a simulation model without the use of the computer.
- Simulation models are much more flexible in representing systems than their mathematical counterparts.
- Important applications of Monte Carlo method are found in waiting line problems, formulating maintenance policies, determining the inventory level etc. Though the process of general simulation can be carried on physical (or iconic), analog (or schematic) and mathematical (or symbolic) models, in the subject of mathematics or any of its offshoots like operations research, it is only the mathematical model that is what is implied.
- A random variable takes on different values as a result of the outcomes of a random experiment. In other words, a function which assigns numerical values to each element of the set of events that may occur (i.e., every element in the sample space) is termed a random variable.
- If in a bivariate distribution the data is quite large, then they may be summed up in the form of a two-way table. In this for each variable, the values are grouped into different classes (not necessary same for both the variables), keeping in view the same considerations as in the case of univariate distribution.

4.12 KEY TERMS

- **Simulation:** A representation of reality using a model or other device that reacts in the same manner as reality under a given set of conditions and is an imitation of a reality.
- **Random Number:** It refers to a number assigned to a random variable following uniform probability density function.
- **System:** A collection of entities that act and interact toward the accomplishment of some logical end.

NOTES

- **Event:** A situation that causes the state of the system to change instantaneously.
- **Event list:** A list in which all the information regarding change of state is preserved.
- **Pseudorandom Numbers:** These random numbers are generated by some deterministic process but have already qualified the predetermined statistical test for randomness.
- **Monte Carlo Technique:** A technique of simulation in which a series of random numbers are used to create statistical distribution function and used where mathematical representation of a problem is not found adequate.

4.13 QUESTIONS AND EXERCISES

Short-Answer Questions

1. When simulation is used? What is done in a simulation?
2. Differentiate between a random number and a pseudorandom number.
3. When can we resort to the Monte Carlo technique?
4. Differentiate between a deterministic and a stochastic simulation model.
5. How simulation is initiated?
6. What is fixed-increment time-advance method?
7. Who gave the code name Monte Carlo?
8. What can you do using the Monte Carlo method?
9. Can simulation be applied to every situation? When can it not be applied?
10. What are Bernoulli trials?
11. What are the significant characteristics of binomial distribution?
12. What is CPF?

Long-Answer Questions

1. Explain simulation and its various types.
2. Do you agree that simulation is a modern day need? Give reason for your answer.
3. What is a random variable? Why is it used? Explain with the help of an example.
4. Explain the methodology of the Monte Carlo simulation techniques.
5. Describe the role of simulation in management process.
6. Discuss Monte Carlo simulation with reference to queuing theory, inventory control and production line.
7. The following data is observed in a tea serving counter. The arrival is for one minute interval.

No. of Persons Arriving	0	1	2	3	4	5
Probability	0.05	0.15	0.40	0.20	0.15	0.05

The service is taken as 2 persons for one minute interval. Using the following random numbers simulate for 15 minutes period.

09, 54, 94, 01, 80, 73, 20, 26, 90, 79, 25, 48, 99, 25, 89

Calculate also the average number of persons waiting in the queue per minute.

NOTES

8. A special purpose drill bores holes having a mean diameter of 1 cm. The process is normally distributed. Simulate a sequence of 10 diameters if the standard deviation of the process is 0.002 cm.
9. At a toll office a sample of 100 arrivals of vehicles gives the following frequency distribution of the inter-arrival and service time (calculate as minutes).

Inter Arrival Time	*Frequency*	*Service Time*	*Frequency*
1.0	2	1.5	10
1.5	5	–	–
2.0	9	2	22
2.5	25	–	–
3.0	22	2.5	40
3.5	11	–	–
4.0	10	3.0	20
4.5	6	–	–
5.0	3	3.5	8
5.5	2	–	–

There is a clerk at the office. Simulate the process for 20 arrivals and estimate the average percentage vehicle waiting time and average per cent idle time of the clerk.

10. Describe Bayes' theorem with the help of an example.
11. Analyse the types of discrete distributions.
12. Generate a sequence of 5 two-digit random numbers by employing the following methods:
    (*i*) mixed congruential method
    (*ii*) multiplicative congruential method
    (*iii*) Additive congruential method.

    The recursive relation for the mixed congruential method is $r_{i+1} = (21r_i + 53)$ (modulo 100).
13. Bright Bakery keeps stock of a particular brand of cake. Previous experience indicates the daily demand as given here:

Daily Demand	0	10	20	30	40	50
Probability	0.01	0.20	0.15	0.50	0.12	0.02

Consider the following sequence of random numbers:

48, 78, 19, 51, 56, 77, 15, 14, 68, 09.

Using this sequence, simulate the demand for the next 10 days. Find out the stock situation if the owner of the bakery decides to make 30 cakes everyday. Also estimate the daily average demand for the cakes on the basis of simulated data.

NOTES

14. A confectioner sells confectionery items. Past data of demand per week (in hundred kilograms) with frequency is given below:

Demand/Week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using the following sequence of random numbers, generate the demand for the next 10 weeks. Also find the average demand per week.

35, 52, 90, 13, 23, 73, 34, 57, 35, 83, 94, 56, 67, 66, 60

15. The manager of a bookstore has to decide the no. of copies of a particular tax law book to order. A book costs ₹ 60 and is sold for ₹ 80. Since some of the tax laws change year after year, any copies unsold while the edition is current must be sold for ₹ 30. From past records, the distribution of demand for this book has been obtained as follows:

Demand (No. of Copies)	15	16	17	18	19	20	21	22
Proportion	0.05	0.08	0.20	0.45	0.10	0.07	0.03	0.02

Using the following sequence of random numbers, generate data on demand for 20 time periods (years). Calculate the average profit obtainable under each of the courses of action open to the manager. What is the optimal policy?

14, 02, 93, 99, 18, 71, 37, 30, 12, 10,

88, 13, 00, 57, 69, 32, 18, 08, 92, 73

16. A Company has a single service station which has the following characteristics: The mean arrival rate of customers and the mean service time are 6.2 minutes and 5.5 minutes, respectively. The time between an arrival and its service varies from one minute to seven minutes. The arrival and service time distributions are given below:

Time (Minutes)	*Arrival (Probability)*	*Service (Probability)*
1–2	0.05	0.10
2–3	0.20	0.20
3–4	0.35	0.40
4–5	0.25	0.20
5–6	0.10	0.10
6–7	0.05	—

The queuing process starts at 11 a.m. and closes at 12 p.m. An arrival moves immediately into the service facility if it is empty. On the other hand, if the service station is busy, the arrival will wait in the queue. Customers are served on the first come, first served basis.

If the clerk's wages are ₹ 6 per hour and the customer's waiting line costs ₹ 5 per hour, would it be economical for the manager to engage the second clerk? Use Monte Carlo Simulation Technique. Use the following strings of random numbers for 20 runs.

64, 04, 02, 70, 03, 60, 16, 08, 36, 38, 07, 08, 59, 53, 03, 62, 36, 27, 97, 86;

30, 75, 38, 24, 57, 09, 12, 18, 65, 25, 11, 79, 61, 77, 10, 16, 55, 52, 59, 63.

17. A Company manufactures around 200 mopeds per day. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopeds to 204 mopeds, whose probability distribution is as follows:

Production Per Day	*Probability*
196	0.05
197	0.09
198	0.12
199	0.14
200	0.20
201	0.15
202	0.11
203	0.08
204	0.06

The finished mopeds are transported in a specially designed three storeyed lorry that can accommodate only 200 mopeds. Using the given 15 random numbers viz., 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10 simulate the process to find out (*i*) What will be the average no. of mopeds waiting in the factory? and (*ii*) What will be the average no. of empty spaces on the lorry?

18. A bookstore wishes to carry 'Ramayana' in stock. Demand is probabilistic and replenishment of stock takes 2 days (i.e., if an order is placed on 1st March, it will be delivered at the end of the day on 3rd March). The probabilities of demand are given below:

Daily Demand	0	1	2	3	4
Probability	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of ₹ 10 per order. The store also incurs a carrying cost of ₹0.50 per book per day. The inventory carrying cost is calculated on the basis of stock at the end of each day.

The manager of the book store wishes to compare two options for his inventory decision:

(*i*) Order 5 books when the inventory at the beginning of the day plus orders outstanding is less than 8 books.

(*ii*) Order 8 books when the inventory at the beginning of the day plus orders outstanding is less than 8.

Currently (beginning first day) the store has a stock of 8 books plus 6 books ordered two days ago and expected to arrive next day.

Using Monte Carlo Simulation for 10 cycles, recommend which option the manager should choose. The two digit random numbers are given below:

89, 34, 70, 63, 61, 81, 39, 16, 13, 73.

NOTES

19. A washing powder manufacturing company wants to study an investment project based on three factors: Market Demand in Units; Contribution (= Sales price – Variable cost) per unit; and investment required. These factors are felt to be independent of each other. The company estimates the following probability distributions:

Annual Demand

Units	*Probability*
20,000	0.05
25,000	0.10
30,000	0.20
35,000	0.30
40,000	0.20
45,000	0.10
50,000	0.05

Contribution Per Unit

₹	*Probability*
3.00	0.10
5.00	0.20
7.00	0.40
9.00	0.20
10.00	0.10

Required Investment

₹	*Probability*
17,50,000	0.25
20,00,000	0.50
25,00,000	0.25

Using Monte Carlo Simulation for 10 runs, estimate the percentage of Return On Investment (ROI)% for each run as defined by ROI% = $\frac{\text{Cash in flow}}{\text{Investment}} \times 100$

Recommend an optimum investment strategy based on model value of ROI%.

Use the following 3 strings of random numbers for demand, profit and investment, respectively.

28, 57, 60, 17, 64, 20, 27, 58, 61, 30

19, 07, 90, 02, 57, 28, 29, 83, 58, 41

18, 67, 16, 71, 43, 68, 47, 24, 19, 97

NOTES

20. The director of finance for a cooperative farm is concerned about the yields per acre she can expect from this year's corn crop. The probability distribution of the yields for the current weather conditions is given below:

Yield in Kg Per Acre	*Probability*
120	0.18
140	0.26
160	0.44
180	0.12

She would like to see a simulation of the yields she might expect over the next 10 years for weather conditions similar to those she is now experiencing.

(*i*) Simulate the average yield she might expect per acre using the following random numbers:

20, 72, 34, 54, 30, 22, 48, 74, 76, 02

She is also interested in the effect of market price fluctuations on the co-operative farm's revenue. She makes this estimate of per kg prices of corn.

Price Per Kg (₹)	*Probability*
2.00	0.05
2.10	0.15
2.20	0.30
2.30	0.25
2.40	0.15
2.50	0.10

(*ii*) Simulate the price she might expect to observe over the next 10 years using the following random numbers:

82, 95, 18, 96, 20, 84, 56, 11, 52, 03

Assuming that prices are independent of yields, combine these two into the revenue per acre and also find out the average revenue per acre she might expect every year.

21. The ABC company is evaluating an investment proposal which has uncertainty associated with the three important aspects: the original cost, the useful life and the annual net cash flows. The three probability distributions for these variables are shown below:

Original Cost

Value (₹)	*Probability*
60,000	0.3
70,000	0.6
90,000	0.1

Useful Life

Period	*Probability*
5	0.4
6	0.4
7	0.2

NOTES

Annual Net Cash Inflows

Value (₹)	*Probability*
10,000	0.1
15,000	0.3
20,000	0.4
25,000	0.2

The firm wants to perform five simulation runs of this project's life. The firm's cost of capital is 15% and the risk free rate is 6%; for simplicity it is assumed that these two values are known for certain and will remain constant over the life of the project.

To simulate the probability distributions of original cost, useful life and annual net cash inflows, use the following sets of random numbers:

09, 84, 41, 92, 65; 24, 38, 73, 07, 04;

and 07, 48, 57, 64, 72, respectively, each of the five simulation runs.

4.14 FURTHER READING

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UNIT 5 APPLICATION OF SIMULATION AND MODELLING

NOTES

Structure

5.0 INTRODUCTION

Simulated environments permit and assists us to assess innovative thoughts and concepts prior to making complex and multifaceted decisions. This analysis technique allows us to deploy various parameters within a system to identify numerous prospects for development and enhancement of the current system. Simulation models are there to provide us graphical representation of information which can be certainly be edited and animated, portraying us the consequences of grasping specific actions. Smearing the generated outcomes in the system certainly assists to accomplish risk and attain success.

Simulated environments are used to simulate new ideas prior to assembly a multifaceted business verdict. This kind of analysis procedure permits you to deploy various parameters, for instance income & expenditures, to determine various prospects for improving in the ongoing operational activities, etc. Simulation models can provide us an explicit presentation or demonstration of statistics that can be amended and animated, displaying us the consequences in case if we adapt specific activities and operational actions. Incorporating the generated consequences to the business assists in managing associated perceived and unnoticed or invisible business risks and impose improved adoptions in management. Simulation can be used in the organization for providing training to staff members, attempting out several situations and conclude the outcomes specific actions shall partake.

In this unit, you will learn about the introduction to application of simulation and modelling, application in management, application in optimization, application in artificial intelligence, application in sociology, application in economics, application in life science, application in database designing and application in computer designing.

5.1 OBJECTIVES

NOTES

After going through this unit you will be able to:

- Understand the introduction of simulation and modeling
- n Descuss the variousapplication of simulation and modeling
- Analysis the application in management
- Learn about the application in optimization
- Illustrate the application in artificial intelligence
- Discuss the various application in sociology
- Describe the various application in economics
- Understand the various application in life science
- Elaborate the application in database designing
- Discuss various application in computer designing

5.2 INTRODUCTION TO SIMULATION AND MODELLING

Simulated environments permit and assists us to assess innovative thoughts and concepts prior to making complex and multifaceted decisions. This analysis technique allows us to deploy various parameters within a system, to identify numerous prospects for development and enhancement of the current system. Simulation models are there to provide us graphical representation of information which can be certainly be edited and animated, portraying us the consequences of grasping specific actions. Smearing the generated outcomes to in the system certainly assists to accomplish risk and attain success.

5.2.1 Application of Simulation and Modeling

Simulation and modelling are used in various environments. The below mentioned list are some of the areas where Simulation and modelling are used extensively to yield better result.

- Modelling in various engineering streams and science – Models in the field of engineering and science are created and simulated to get the outcome of a specific output for research purposes.
- Big Data – Simulation is used for big data to perform trend analysis and other associated analysis.
- Internet of Things (IoT) – IoT are physical devices (TV, Refrigerator, Washing Machines, etc) which consists of sensors, ability in the area of processing, associated and relevant software and other associated technologies using which they can be connected with the internet and exchange information with other devices and systems. Modelling and simulation techniques are used to model these devices and study the behaviour of them when hooked up with the internet.

- High Performance Computing and Network Simulation – Modelling and simulation is used to study the performance during high level of computing. With successful analysis, inference of increase/decrease of computing speed and capacity can be decided. Additionally, simulation is also used for complex network to study the data traffic and take adequate actions as applicable.
- Artificial Intelligence – AI or Artificial Intelligence is the application of technology using which a machine can be enabled to think. Simulation is used to study the visual perception of the system, recognition of speech, decision-making, and translation between various languages.
- Machine Learning - Machine learning is a part of artificial intelligence (AI) which mainly concentrates on the usage of data and algorithms to replicate the way humans learn. During generating the algorithms, simulation is used to check the performance and the correctness of the algorithm.
- Simulation is also used in Supercomputers to check the detonation of nuclear devices and its associated effects for supporting in an improved manner during nuclear explosion.
- Simulators are used to simulate storms and other natural disasters. Simulations deliver precise forecasts of weather based on the enormous arrangement of satellite, radar, and other data available.
- To prepare for successful mission, NASA uses simulation to simulate the planetary surface.
- Simulation is also used in conventional decision support systems.
- Modelling and simulation is used to setup a comprehensible artificial environment.
- Used for research and developments in the area of Systems Theory, Operations research.
- Several pharmacy colleges and medical researchers have included simulation as part of their principal curriculum.
- Simulation is used extensively in the area of Analyses Support and experimentation.
- Simulation is used for procuring, developing and associated testing of specific systems.
- Training and Education – Simulators are used to judge the virtual training environments for providing training to people.
- NASA uses simulators for simulating the orbit modelling in the area of space research.
- Power system uses simulation to judge the behaviour and whereabouts of power grids and other complex power related areas.
- Rule-based modelling – Simulators are used in developing rule-based models.
- Logistics simulation – Simulations are used for optimizing multifaceted and dynamic logistics related process.

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- Simulation in production – It includes modelling solitary production lines, i.e., from the designing the production resources and buffer sizes to the simulating the complete production plant.
- Detailed production planning – Simulation is used for the optimization of initial preparation while taking into consideration the dynamic factors like existing accessibilities or commotions, resources, stocks, reorder level of the facility, etc.
- Emulation - Using simulation, virtual testing is accomplished for observing the software controls.
- Planning the scheduling of machinery- Optimising machine capacity application by minimalizing set-up times and preventing standby- and waiting times.
- Simulation of control station - Optimisation of control approaches by means of simulation.
- Personnel simulation - Support for personnel arrangement and personnel posting.
- Simulating Supply Chain - Modelling and analysis of supply networks via simulation.
- Financial planning – Simulation is used for determining the effect of capital investment related decisions, purchase or manufacturing related decisions and cash flow management strategy related decisions.
- Environment and ecological systems – Simulation is used for pollution control policy for a specific town or city, observing the flood control related policies, simulate the paint shop scheduling for minimizing pollution related strategies.
- Simulation in project planning and control – Simulation is used in determining the portfolio management for new products, dynamic resource related actions to various projects, overlapping resource allocation to projects.
- Communication System – Simulation is used for network designing, designing of client/server related system, switch buffer management, etc.
- Computer application system – Simulation is used to design E-commerce related cloud applications and observe its operational behaviours, designing of wide area network and selections of associated protocol, observation of the dynamic precedence assignments for batch jobs in memory queue.
- Military system – Simulation is used to design the defence system, airlift system, etc. Simulation is also used in the area to schedule flight maintenance.
- Public System Office – Simulation is used to identify the settings of social security offices and used to determine the flow of civil lawsuits. Simulation is also used for planning and scheduling manpower.
- Healthcare – Simulation is hugely used in the healthcare environment. It is used to find the feasibility of hospitals and clinics in a specific geographic area. Simulation is used to study the behaviour of new equipment in a clinic or in a hospital. Simulation is used for generating rotas for nurses and technicians in a hospital across various departments.

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- Transportation and logistics – Simulation is used to determine the assortment of hubs and geographic markets for an airline. Additionally, simulation is used to identify the capacity of airplanes, and scheduling and routing of flights.
- Warehouse and distribution system - Simulation is used to identify appropriate warehouse locations, analyse the warehouse capacity and layout, status about the dedicated against random storage and the sequence of pickup of orders.
- Material handling system - Usage of conveyors or Automatic Guided Vehicles (AGV) within an assembly plants, Capacity and number of Automatic Guided Vehicles (AGV), routing assignment of fork lifters, etc.
- Manufacturing system – Used for designing & laying out of a plant, procurement and acquisition of new machineries, scheduling & controlling of jobs, etc.

5.3 APPLICATION IN MANAGEMENT

Simulated environments are used simulate new ideas prior to assembly a multifaceted business verdict. This kind of analysis procedure permits you to deploy various parameters, for instance income & expenditures, to determine various prospects for improving in the ongoing operational activities, etc. Simulation models can provide us an explicit presentation or demonstration of statistics that can be amended and animated, displaying us the consequences in case if we adapt specific activities and operational actions. Incorporating the generated consequences to the business assists in managing associated perceived and unnoticed or invisible business risks and impose improved adoptions in management.

Simulation can be used in the organization for providing training to staff members, attempting out several situations and conclude the outcomes specific actions shall partake. Simulation software (using which we can develop relevant charts and generate appropriate models) are used to adapt healthier decisions. The significant thing is to refer to standard tools which the service provider or vendor has previously tested with the set of existing data and methodologies to authenticate and validate its throughput in terms of performance. Once the standard simulation tool provides consistent results, it can be used to simulate data to achieve desired output.

Improvement in Processes

Simulation models are used to analyse current ongoing business processes for improvement. A standard simulation model concentrates on a particular characteristic of the business, in the area of manufacturing or finance as well. By simulating all the particulars of how things are working currently, the bottlenecks can be pinpointed and alternative techniques can be identified to improvement the business processes managerial decisions. If situations are properly simulated, it will propose smaller changes which can create better impact for business. Let us consider an example. A hospital has 200 nurses and it operates for 3 shifts. Among the nurses, there are various categories (e.g., experienced, fresher's, learners, etc). To accommodate them in a properly combined manner in the 3 shifts, manual rota becomes cumbersome. To yield an optimum solution, if the model can be replicated via simulation model, it will certainly provide various optimal solutions using which the nurses can be placed in the various shifts efficiently. Simulations models are such efficient that it can also pin point the unnoticed or overlooked factors which, when considered can provide better result.

Outcome Predictions

Spreadsheets are also used to simulate business cases. The What-if analysis with Scenario Manager and Goal Seek is the modules in MS Excel which is used to do so. With this tool various sets of data can be used with the help of one or more formula to discover all possible results. This assists to yield much precise and accurate predictions. For adapting successful management relate decisions, these kinds of simulations are very important. For example, a construction company wants to buy several materials required to construct a fly-over of 2 kilometres in length. This kind of simulation definitely helps the specific company to take decisions on various aspects like procurement related decisions, project management related decisions, etc. Google Docs can also be used to generate a scatter plot along with a trend line which depicts the complete course of the process flow which is referred to while taking management decisions.

Managing Business Related Risk

Manipulating data efficiently empowers an organization to scrutinize how much can be invested or lose business under specific simulated conditions. For example, this can be achieved via Monte Carlo simulation. This is used in casinos to find out chances of winning or losing. By providing the revenue and prices (which can be considered as uncertain inputs), probable output and results can be identified. Spreadsheet can also be used to simulate the flow of cash, evaluate rate of return and risk associated in introducing new products, identifying the risk related to the fluctuations of exchange rate or identify various strategies related to new investment.

Deciding on Actions

Spreadsheets or computer simulated models are used to help organization in taking various management decisions. Known data and results are used as inputs to the simulation system for calibrating the required model to provide precise outcomes. For example, simulation can be used to conclude how long a production run will generate maximum revenue. Factors like authentic costs of production, pricing and costs of overproduction should be feed into the simulation system and let the model will compute the likely profit for different production runs. This result will assist the management to decide on the quantity of items the organization should produce.

Improving of Forecasts

Simulations can assist you in refining business forecasts. This is accomplished by the simulator by referring to historical data. With the correct of data, the model can simulate and yield forthcoming values for the future projects. Various use cases can be generated by variable set of parameters. For example, if historical sales data can be fed into the model, the simulator can deduce sales forecasts linearly for future. More sophisticated models can also generate future sales data including the disparities during holiday seasons and peak seasons. Simulators can forecast volumes of sales in case price of products are reduced. Simulations can also be used to predict the sales figures against the effect of the product advertisements. Using these predicted data, the organization can take decisions in terms of cost enhancement or reduction. At the same time the organization can decide whether mode advertisements are required or not and in which specific localities.

Exploration of Likely Scenarios

Simulations are also used for evaluating various scenarios to make managers of an organization aware of probable events. During analysing markets in terms of sales, managers often require to compare their products against the competitive products. This can be easily done via simulation. For example, a competitor for the sake of increasing sales, could bring down the prices of their product or announce products with enhanced attributes. Proper simulation can analyse this kind of situation and conclude what should be done to increase sales of a product which has not been enhanced in terms of its features. Simulations can be used to identify effect before implementing new regulations within a state or a country. Simulations are used to assess theoretical events in in the area of sales and revenue. Based on the analysis, corresponding managers can take decision on approaches to counter uninvited effects and decrease the chance of exposure and associated risk for their organization.

Improving Employee Performance

Simulations are used to coach and groom employees for addressing intricate business environments successfully. Using modelling, decisions can be drawn in terms of forecasting which trainings are required by the employees. Accordingly, training can be provided via simulation. For example, if an organization needs to promote quality awareness, trainings can be simulated along with associated examinations for the subject. Change controls can also be simulated using simulators which can forecast the effect of the change.

5.4 APPLICATION IN OPTIMIZATION

Simulation optimization is defined as the procedure to identify the unsurpassed input variables from all likelihoods of data sets without specifically assessing every single data. The primary purpose of simulation optimization is to curtail down the resources expended while make best use of data attained in a simulation trial.

Simulation optimization is also known as Simulation-based optimization. It is used to embed various optimization methods and practices within a simulation model and scrutinize the situation. Normally, the fundamental simulation model is considered to be stochastic. Hence the unbiased purpose should be assessed using statistical estimation methodologies which is known as “output analysis” in simulation practice.

Post a specific system is modelled using mathematical functionalities, the computer power generates data about its associated behaviour. “Parametric

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simulation" is used to expand the throughput of a specific system. In this methodology, the input of every variable is adjusted with supplementary parameters maintained to be constant and the impact on the strategically purpose is detected. This method certainly consumes a huge amount of time. To attain optimum solution with least calculation and time, the hindrance is resolved in a iterating manner where every repetition makes the procedure move closer to the optimum solution. This kind of method is known as 'numerical optimization' or 'simulation-based optimization'.

Significant characteristic of optimization modelling (mathematical) is the usage of mathematical equations and procedures to generate models that works in the similar way as the process which is modelled. This can be applicable to a business model which is very similar to a CAD model. CAD models are created to represent a physical model along with its corresponding strengths and rigidities.

Simulation optimization and modelling can be achieved with physical and mathematical models. For example, to identify the optimum preparation of numerous aero mechanisms to diminish struggle at high speed or to intensification down force, Formula 1 racing car team members form precise scale models for the cars

Managers dealing with supply chain in corporates, uses optimization modelling to design supply chain related network to identify the optimal combination of suppliers, sites, logistics and manufacture for optimum supply.

There is a difference between optimization modelling and simulation Optimization modelling propose precise approaches, policies and strategies. Simulation is used to improve situation development and assists in answering questions related to what-if.

Precise simulation–based optimization approaches can be selected according to the Figure 5.1.

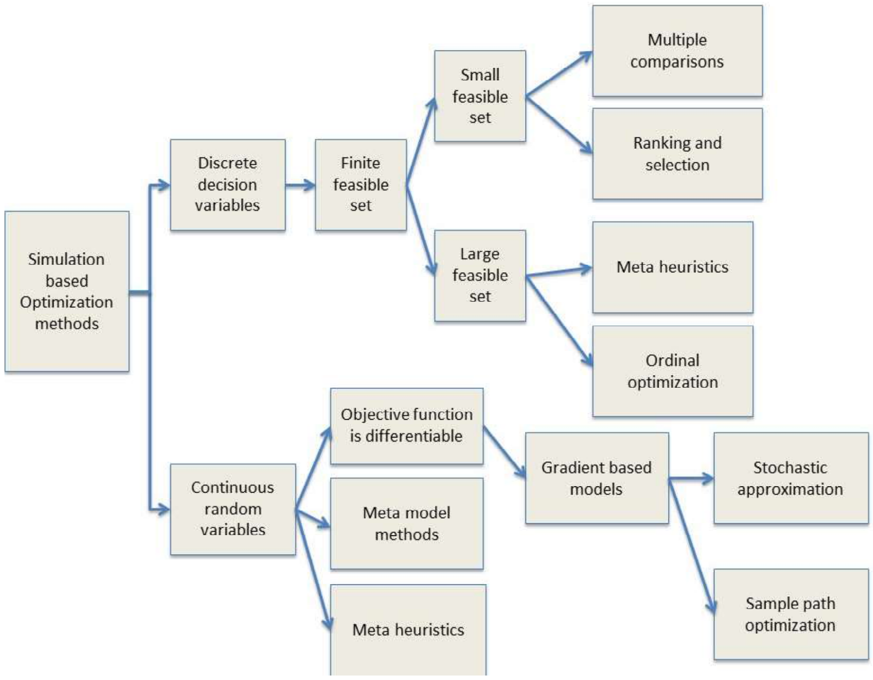

Fig. 5.1 *Types of Simulation-Based Optimization in Accordance to Variable Types*

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Optimization primarily is dealt extensively in 2 main areas of Operations Research (OR). They are as follows:

Optimization Parametric (static) – The main intend is to identify the values of the parameters, which are "static" in nature for all possible conditions, with the objective of increasing or decreasing a functionality. For this, linear programming is used. Simulation for this assists a lot, in case, noises are existing in the parameters or assessment of the difficulty would request for tremendous computing time, as the situation is very complex in nature.

Optimization Control (dynamic) – This kind of optimization is only used in the area of computer science and electrical engineering. The optimum regulator is per state basis and the outcome gets altered in each of the states. Normally mathematical programming or dynamic programming is used in this area. Simulation produces random samples and resolves compound and large-scale issues.

Some significant methods in simulation optimization are as follows:

Statistical Ranking and Selection Methods (R/S)

This method is considered in resolving issues where the substitutes are static and identified. The simulation here is used to evaluate the performance of the system. The simulation uses approaches like "Indifference zone approaches", "Optimal computing budget allocation" and "Knowledge gradient algorithms".

Response Surface Methodology (RSM)

In this procedure, the aim is to identify the association in between the input variables and the response variables. The process initiates with attempting to adapt a linear regression model. In case value of P is identified to be low, then a higher degree quadratic polynomial needs to be employed. The way of identifying a better association between input and response variables can be accomplished for each simulation test. "Response surface method" is used to identify the most appropriate input variables which results anticipated consequences for the response variables.

Heuristic Methods

This method alters accurateness by speediness. The main aim is to identify a faster method for solving a problem. Normally this process identifies the local optimal value as an alternative to the optimal value. The values are very near to the concluding solution. Tabu search and genetic algorithms are some of the examples for this method.

Stochastic Approximation

This method is used is used when the function fails to be directly calculated. It is only approximated using noisy observations. In this case, this method checks for the extrema of the following function

$$\min_{x \in \theta} f(x) = \min_{x \in \theta} E[F(x,y)]$$

where

y is the noise and is considered to be a random variable x is the minimizing parameter of f(x)

θ is x parameter's domain

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Derivative-Free Optimization Methods

It is purely related to mathematical optimization. This method is put into place to a specific problem related to optimization when its corresponding derivatives are absent or untrustworthy. Derivative-free methods founds a model which is on the basis of sample function values. It can also be straight away drawn from set of functions without manipulating a comprehensive model. Since this method does not require any derivatives, it can never be compared to the derivative-based methods.

For unconstrained optimization problems, the associated form is as follows:

$$\min_{\mathrm{x}\in\mathbb{R}^n} f(\mathrm{x})$$

Dynamic Programming

This process normally addresses circumstances where conclusions are derived through various stages. The solution of this issue deals with the current and forthcoming costs.

This kind of model consists of two attributes:

- It has a separate time dynamic system.
- The cost function is preservative on top of time.

For discrete features, dynamic programming has the form:

$$x_{k+1} = f_k(x_k, u_k, w_k), k = 0, 1, \ldots, N-1$$

where

k is the index of discrete time.

x_k denotes the state of time k, it consists of the previous data and formulates it for upcoming optimization

u_k denotes the control variables

w_k denotes the random parameter

Cost function is represented as:

$$g_N(X_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, W_k)$$

$g_N(X_N)$ is the cost at the end of the process.

Here the cost cannot be optimized expressively. Hence the corresponding expected value can be used.

$$E\{g_N(X_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, W_k)\}$$

Neuro-Dynamic Programming

This type of programming is similar to dynamic programming. The exception is that dynamic programming supports the notion of approximation architectures. It amalgamates Artificial Intelligence (AI), simulation-base algorithms and functional approach methodologies. The word "Neuro" in is heavily used in the arena of Artificial Intelligence (AI). It is about the study of the process of making better-quality conclusions for the forthcoming by using built-in mechanism which is on the basis of the present behaviour. The foremost significant portion of neuro-dynamic programming is to generate a skilled neuro network for the optimum solution.

In modern business arena optimization using simulation and modelling is frequently used in the following fields:

- Capacity Planning
- Commodity trading & associated logistics
- Mixing of products
- Profitability of clients
- Planning of production
- Planning of sales
- Planning related to operational activities
- Supply Chain
- Optimization of logistics
- Optimization in the area of promoting trade

Following are some the industry where optimization using simulation and modelling is used

- Products building
- Chemicals
- Consumer Packaged Goods
- Manufacturing
- Metals and Mining
- Oil & Gas
- Paper and Pulp

5.5 APPLICATION IN ARTIFICIAL INTELLIGENCE

Artificial intelligence known as AI in short is a vast area of study in computer science which deals with creating smart computers which are proficient in accomplishing activities which normally necessitates intelligence of human being.

While defining the attributes of artificial intelligence, scientists like Norvig and Russell defined AT as:

a. Thinking like human being

b. Thinking sensibly and realistically

c. Acting like human being

d. Acting sensibly and realistically

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#a and #b deals with the thought processes and reasoning. #c and @d deals with the behaviour.

Patrick Winston, who was the "Ford professor of artificial intelligence and computer science" at MIT, has described artificial intelligence as "Algorithms enabled by constraints, exposed by representations that support models targeted at loops that tie thinking, perception and action together."

Examples of artificial intelligence applications are as follows:

- Google uses AI during searching
- Software used for image recognition
- Vehicles supporting auto-driving
- Watson from IBM
- Siri, Alexa and other smart applications related to personal assistance
- Robo-advisors
- Typical bots
- Used to filter spams in emails
- Recommendations by Netflix
- Google's AI-powered forecasting (e.g., Google Maps)
- Applications related to rides (E.g., Uber cab, Ola cab, etc)
- Auto piloting technology uses AI (normally in commercial aircrafts)
- AI is used in plagiarism checking
- AI is used for facial recognition
- Voice-to-text related applications used AI extensively
- AI is used for fraud preventions and fraud protection related activities.
- Application such as "Facebook Watch" uses AI to specifically identify the search items.
- Facebook uses AI for Friend recommendation
- AI is used extensively in robo-advisors which is used for advising on the basis of specific inputs.
- AI is used for proactive healthcare management related activities. For example, pathologists and biomedical professionals uses various AI powered machines during performing pathological tests. These machines are able to forecast degree of an identified disease to a greater extent.
- There are various AI powered applications which are able to forecast financial investing on the basis of the historical trend of an individual.
- Virtual travel booking application uses software which are driven by AI.
- AI is used for monitoring social media for various purpose e.g., national security, tracing criminals by police, etc.

- In the area of healthcare all applications and devices use AI for its specific purpose e.g., Telemedicine, Abetted Diagnosis, Robot-assisted surgery specifically in the area of Colorectal Surgery, General surgery, Gynaecologic surgery, Cardiac surgery, Endometriosis, surgery in relation to head & neck (Transoral), Thoracic surgery, Urologic surgery, etc, monitoring of critical statistics, assistance in virtual nursing, research and study, management chronic care, etc
- In business, AI is used in e-commerce application. It is used for appropriate references, AI chatbots, filtering out spam, search optimization and management in supply-chain.
- Even in the area of human resources related applications AI is used. Specifically in the area of establishing of work culture within the organization, manpower hiring process which normally uses Natural language search (NLS).

Artificial intelligence functions by amalgamating huge volume of data with speedy, iterative-aspect processing and intelligent algorithms, permitting the associated applications to gain knowledge repeatedly and intermittently from trend/trends or pattern/patterns or characteristics in the array of data. Artificial intelligence is a comprehensive arena of study that comprises of various philosophies, concepts, thoughts, perceptions, approaches, procedures, logics and technologies. This includes the following foremost subareas as well:

Machine learning mechanizes or automates the constructing of the analytical model. This method uses approaches from the field of neural networks, statistics, operations research and physics to identify the veiled understandings and acumens in the array of data without evidently being programmed for what needs to be done or where to find or what to find.

A **neural network** is a category of machine learning process which comprises of interrelated or interconnected or unified components or units (e.g., neurons) which processes information by replying to exterior inputs, depending data between each component or unit. The process needs various passes at the data level to identify associated acquaintances and generate sense from the undefined array of data.

Deep learning uses enormous neural networks with various layers of processing units, captivating benefit of advancements in the power of computing and enhanced training methods to study multifaceted patterns in enormous amounts of data. Usual applications consist of recognition of images and speech.

Computer vision trusts on pattern recognition and deep learning to distinguish what is there in a image or video. When machines can process, analyse and comprehend images, they can apprehend images or videos in real time and construe its corresponding environs.

Natural language processing (NLP) is the capability of computers to analyse, comprehend and produce human language which includes speech as well. The subsequent stage of NLP is natural language interaction, which permits humans to interact with computers by means of usual, day to day linguistic to accomplish certain tasks.

Additionally, numerous technologies empower and support Artificial intelligence. They are as follows:

Graphical processing units are most crucial to AI as these entities deliver substantial computing power which is an essential and mandatory for iterative processing. Training neural networks necessitates big data along with huge compute power.

The **Internet of Things** produces enormous volume of data from associated machineries and apparatuses, most of which is not analysed. Automating models powered with AI permits us to utilise supplementary of it.

Advanced algorithms are coded and associated in innovative manner for analysing supplementary data quicker and at numerous levels. This sensible, logical, rational and intelligent processing is the basic principle in recognizing and forecasting infrequent, occasional, sporadic and exceptional events, comprehending multifaceted systems and optimizing exclusive situations.

APIs, or application programming interfaces, basically are portable packages of code that enables to append and enhance AI related operation to a prevailing items and software applications. APIs are used to complement image recognition proficiencies to security systems related houses and Q&A functionalities which define data, produce subtitles and headlines, or introduce appropriate and applicable patterns and interpretations in data.

Artificial Intelligence (AI) is basically the facility or capability of an artificial machine to perform in an intelligent fashion. Logic Programming is a technique that is used to permit to enhance a computer to purpose as it is advantageous for knowledge depiction. In logic programming, logic is used to signify knowledge and conclusion or deduction.

The logic which is used to characterize knowledge in logic programming is clausal form which is certainly a subcategory of first-order predicate logic. It is used since first-order logic is satisfactorily comprehended and capable of expressing or indicating all associated computational glitches. Knowledge is controlled by means of the resolution inference system which is obligatory for demonstrating theorems in clausal-form logic. The diagram (Fig. 5.2) below displays the kernel of logic programming.

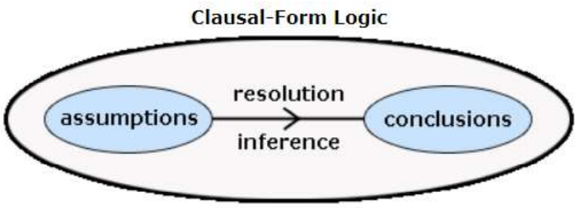

Fig. 5.2 *The Essence of Logic Programming*

Prolog i.e., PROgramming in LOGic, is a declarative programming language which is based on the thoughts and philosophies of logic programming. The indication of Prolog was to construct logic appear like a programming language and permit it to be regulated by a programmer to improve and progress the research for theorem-proving.

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Artificial Intelligence methods, e.g., evolutionary algorithms and influence diagrams, are normally used to identify the type of incremental degradation in the automated procedure of simulation beforehand it growths up to be a trash.

Artificial Intelligence methods such as procedural heuristics and constraint resolution are used to direct the simulation in determining at what time and if, involuntary defensive strategies can be superseded, in scheduling the restart and workaround.

We can see that building up AI enabled logics are very complex. Hence prior to setup the coding part the model should be tested using simulation to get a perfect picture of whether the model intended is perfect or not.

On the other hand, Artificial Intelligence is also used in simulation tool.

	Application Areas	Success Stories
Machine Learning	• Workflow automation • Decision support • Anomaly detection	• Consumer fraud behavior anomalous activity detection using unsupervised methods • Deals asset reclassification automation support
Deep Learning	• Unstructured data analysis for audio, image, and text • Automation and learning systems	• Audio analysis used to identify sales call features and assess traits of effective conversations • Deep reinforcement learning used to learn optimal operating policies for a rideshare business
Natural Language Understanding	• Information extraction, workflow automation, and text generation • Insights from unstructured data • Chat bots and IPA	• Tax workflow automation that could save the client ~$65M annually • Customer contact center complaint analytics platform • Comprehensive chat bot study
Data at Scale	• Industrial IoT • Streaming and real time data • Large scale data processing architecture	• Predictive maintenance applying ML on oil/gas pump sensor streaming data in a scalable manner
Simulation	• Operations analysis • Strategy support • Consumer behavior modelling • New market entry analysis	• Designed a go to market strategy for a new personal mobility service • Patent for autonomous vehicle resource management in a rideshare environment

***Fig. 5.3** Five Key Areas in AI*

Systems which are already using AI, e.g., digital supply chains, smart factories, and other industrial procedures which established industry 4.0, shall require to comprise Artificial Intelligence in their simulation models. For instance, with the digital twins (AL introduction paper) and what-if analysis systems, the Artificial Intelligence constituents can be unswervingly implanted in the simulation model to enable testing and prediction.

Another significant prospect in simulation modelling is the application of Artificial Intelligence to optimize and calibrate. Huge number of parameters are used to explore all its probable permutations in an agent-based system and it requires unreasonable run times. Machine learning and intelligent sampling can be certainly used to fabricate meta-models which, in turn can produce huge increase in speed for large-scale agent-based models.

For deep learning, AI components can be generated to substitute the rule-based models. This is certainly achievable when taking into consideration the human behaviour and decision making. These components which are associated to deep learning can appropriately be used in simulation models to reproduce the production environment. On the other side, simulation models can also be used to fine tune the AI components. By producing data sets required in fine tuning the neural network, simulation models can also be a prevailing instrument when installing deep learning in the production environment.

Use cases for AI Enabled Simulations

Case 1: Models are used as a substitute for inputs that characterizes abstracted behaviour within a model

Input parameters are used in a simulation model for defining certain guesstimated or assessed behaviour which is completely on the basis of the causal rules existing in a production environment (e.g., delay time, arrival rate, etc). They are frequently modelled as univariate random variables or, in certain situations, as a random vector with multivariate probability distributions. Simulations and modelling procedures powered by AI is an appropriate use for this stated case.

Case 2: Using AI powered models to approximating the performance of components within the simulated system.

When specific mechanisms or elements within a simulation model are extremely multifaceted as it requires huge number of details, a AI enabled model is normally used to conclude the feasible behaviour. An example of this situation can be a substantial apparatus which cannot be easily modellable by normal simulation methodologies and is possible to model with the data sets which can be easily approximated with the help of machine learning.

Case 3: Including any prevailing, deployed models into the respective simulated setting for augmented accurateness.

A simulation model ideally reproduces the defined rules of a production environment. This is true for AI enables simulators as well. Rules and behaviours which are the unswerving consequence of a system where AI solutions is installed, that particular model should also be introduced within the simulation.

Case 4: Testing of various effects and impacts of an AI solution over an existing system's overall performance prior to installation

To check the performance of a real system when AI related components are deployed on top of it, it is essential to test the situation thoroughly. This testing has to be simulated properly as it might lead to various kind of bottlenecks and ripples on the existing system. Hence the AI enabled model should be made and simulation should be done in a perfect manner to replicate the existing system. This simulation can be virtual but will yield proper feedbacks and should forecast situations required for fine tuning the existing system.

Neural Networks is a branch of AI and it also offers enhanced performance on top of conservative technologies in areas of Machine Vision, Detection of robust pattern, Filtering of signals, Virtual Reality, Data Segmentation, Data Compression, Data Mining, Text Mining, Artificial Life, Adaptive Control, Optimization and Scheduling, Complex Mapping, etc. To do a successful implementation in the said areas, AI enabled simulation is a mandatory.

Fuzzy sets are also used in tremendous amount for modelling and simulation. To address such complex situation and opt to make things work perfectly, appropriate situations need to be simulated via AI enabled simulation.

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Computer-Aided Engineering (CAE) amplified by AI is catering automobile and aircraft constructers the capability and aptitude to innovate machine learning-guided perceptions, discover innovative resolutions to composite design related difficulties by means of physics and AI-enabled workflows and attain superior invention via association and design related convergence. These Design generation, design exploration and design optimization are only successfully possible by means of successful AI enabled simulations and high-fidelity modelling.

Check Your Progress

1. What do you mean by the simulated model?
2. Why simulation environments are used?
3. Define simulation optimization.
4. What are two main areas of operations research in optimization?
5. State about the heuristic methods.
6. Define artificial intelligence.
7. How scientists like Norvig and Russell defined AT?
8. Write some examples of artificial intelligence.
9. What is natural language processing?

5.6 APPLICATION IN SOCIOLOGY

Sociology is the area of human social relationships and organizations. The range of this study varies from crime to religion, from households to the states, from the variation of various community and social levels to the common and collective opinions of mutual ethos, and from social constancy to fundamental alteration in entire civilizations and cultures. The purpose of sociology is to unify these varied topics and comprehend how human activities and awareness both form and are formed by adjacent cultural and associated society constructions.

Sociologists frequently established model in regards to various social processes for the purpose of interactions among variables. This is accomplished using Agent-Based Models (ABMs). These agent-based models (ABMs) demonstrate how easy and foreseeable local interactions can produce conversant but inscrutable global designs, such as the dispersal of information, development of norms, organization of settlements or contribution in cooperative action. Developing social patterns can also be noticed in an unpredictable manner and then suddenly get transformed or disappeared, which similarly occurs during revolutions, crashing of market, fashions, and feeding furies.

Topics cover in the study of sociology consists of the following:

- Various theories related to Sociology
- Relation between Society and religion
- Sociology and politics
- Research related to sociology and its corresponding methodologies

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- o Analytical, Empirical, Comparative and exploratory research standpoints
- o Positivism and its associated analyses
- o Qualitative Research Methodologies: Ethnography, Case Studies, Interviews, Narratives
- o Quantitative Research Methodologies: Statistical Methods, Survey, Evaluative approaches

- Sociology in the area of development
- Sociology and gender
- Society, Social Change, Human culture, Social Stratification
- Area-based findings
- Laws related to sociology
- Social establishments
- Sociology in the area of education
- Sociology association with health and hygiene
- Industry and associated society
- Sociology in the area of mass media
- Economic Sociology
- Environment and society
- Rural Sociology

So, we can see that sociology is the methodical study of people, establishments and associated social phenomena. It is a complex study. Conclusion of theories and principles/ philosophies / ethics, etc., requires a huge amount of research dealing with enormous set of numbers and mathematical or statistical methodologies. A sample flow of research in the area of sociology is depicted in Figure 5.4.

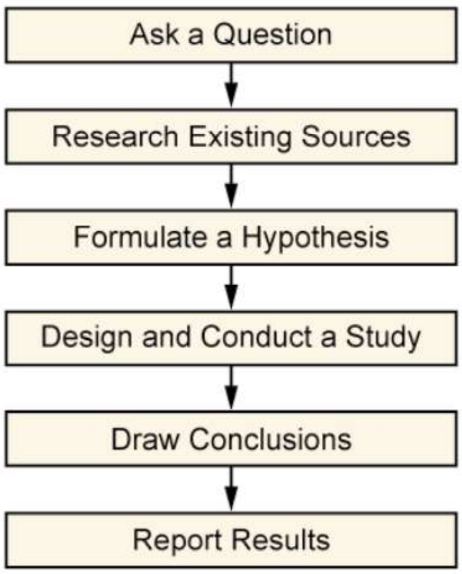

Fig. 5.4 *The Scientific Method is an Essential Tool in Research.*

Sociologists uses scientific technique not only to assemble data but to analyse the collected data to conclude a meaningful or philosophy or concept. They intentionally employ scientific logic and objectivity to conclude a meaningful notion.

Simulation has an extended trend in arena of functional attention to sociology, ranging from prior to Forrester's model of overpopulation to up-to-date approaches based on complicated philosophy or distributed artificial intelligence. Additionally, advanced simulation ability caters certain improvements, specifically the modelling of macro-micro relates to multifaceted concepts which deals with linguistics or mathematics.

Specifically, simulation has become extremely popular in anthropological and historical demography, where simulation has been used extensively in reconstructing various procedures and processes that were only moderately detected. Examples in such conservative demography comprise of Dyke and MacCluer (1975) and, Pennec (1993). Roth (1981) and Whitmore (1991) has focused on the historical reconstruction of the change in human population, Doran (1970) and Wobst (1974) represented initial usage of simulation in archaeology.

With the advent in computer science and its associated areas both qualitative and quantitative aspect of the simulation are also taken into consideration. Advancement in the area of object-oriented programming, neural networks and AI has helped scientists to model and simulate research activities for sociology.

Object-oriented programming has enabled scientists to model and simulate situation in terms of computationally and conceptually and specifically this technology is used to model and simulate nested structures e.g., societies which consists of groups and groups containing human being.

Neural networks are used for creating models of systems or allied agents along with the aptitude to study from the respective environment.

The 'artiûcial life' has established the 'cellular automata' model. It is an extremely helpful standard for modelling humans associated in a spatial structure), and the 'distributed artiûcial intelligence' approach in which systems consists of artiûcially intelligent agents. Which is used to study problems of a chaos and complexity theory.

System dynamics was used for simulation for generating fascinating results. System dynamics is a modelling system, where systems contain of an array of numbers which can expand or shrink, and the rate of expansion and reduction are influenced by 'feedback loops' from the considered stock and other stocks or factors as well, in a hypothetically multifaceted method. Once a simulation generates an output, it is further analysed for a conclusion. Forrester's famous world model (Forrester, 1973) consists of five such factors or stocks:

- Population
- Natural resources
- Capital investment
- Capital investment in agriculture
- Pollution

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These factors or forms are connected with the help of feedback loops. For example, the rate of death is influenced by the population, pollution, and comfort. Both production and population give rise to pollution, and the absorption rate of pollution is certainly a diminishing factor or function of the level of pollution.

There is a huge number of features which is very much relevant for simulating various aspects of sociology. Among these are the beneûts of establishing of formal statement, hypothesis and manipulating or reshaping theories along with solutions for new approaches. Simulation is also used to solve central problems of sociology i.e., the agency structure duality and the relationship in between the macro and the micro stages. The art of converting a theory or notion into a simulation need assembling the whole thing unambiguously, including the internal discrepancies and interruptions.

What is more enthusing about simulation's capability for sociology is the process in which it addresses the central problems in the disciplined manner and plots the solution against them.

In sociology, research as usual gets initiated with a problem statement or a concern related to societal realism. For example, problem statement in sociology may be like identifying the major factors favouring law-breaking, identifying the distinguishing characters of youthful squads in a large city, identifying the reasons of mounting loneliness of individuals or finding out the internal forces of immigrant public.

After articulating the problem, a corresponding theory is established and then checked through a simulation process, (considering the component of analysis i.e., global cultures, social categories, groups, organizations, entities) the feasible solution. While using simulation, selection to the finest approach towards resolution, associated appropriate methodologies, procedures used for collecting, organizing, describing of data are some of the most important factors which should be considered for producing error-free results. Additionally, data collection, is considered to be the indispensable facet of sociological research.

Social simulation is defined as a type of strategy or procedure which is used to learn the various social dynamics. This process uses computers to simulate social systems. Social simulation enables an individual to view solutions in a systematic technique with probable and feasible outcomes.

Social simulations are of four types. They are:

- System level simulation.
- System level modelling.
- Agent-based simulation.
- Agent-based modelling.

Social simulation comes under the purview of computational sociology which uses computational methods to analyse social characteristics. It encompasses the understanding of social agents, various types of interaction among these associated agents and the associated effect of these corresponding interactions on the social cumulative.

System Level Simulation (SLS) is the first-born form of social simulation. Takes care of the system as an unabridged manner. This method uses a vast array of data to identify what should ideally occur to the society and its associated members in case there is a change in a specific variable or what will happen in case there is an introduction of new variable. For example, if NASA opts to accomplish a system level simulation to find the optimal factors for implementing a cost-effective research methodology they have to go through this type of simulation. This simulation will provide all probable virtual likelihoods in developing and implementing a safety process.

System Level Modelling (SLM) targets to precisely forecast and express or suggest various amount of actions, behaviours or other hypothetical options of person, object, construct, etc within a specific system by means of a huge array of mathematical equations and computing power in form of various appropriate models.

Agent-Based Social Simulation (ABSS) involves modelling various societies post artificial agents, (which differs on scale) and places them in simulated society to scrutinise the behaviours of the agents. From this array of information, it is very much feasible to note the responses of the artificial agents and interpret them into the consequences of non-artificial agents and simulations. The 3 foremost area in agent-based social simulation are

- agent-based computing
- social science
- computer simulation

Agent-based computing (ABBS) is related to the fabricating of the associated model and agents. Simulation is a portion of the simulation of the agents within the model and the associated consequences. The social science is a combination of sciences and social portion of the model. This is the area where social occurrences are generated and hypothesized. The foremost aim of ABSS is to generate models and tools for agent-based simulation of social occurrences. With ABSS, various consequences for occurrences can be discovered which may not be possible in real life. ABBS provides appreciated information on the society and the consequences of social proceedings or occurrences.

Agent-Based Modelling(ABM) is a system where group of agents self-reliantly communicate on specific networks. Every agent is accountable for a unique kind of behaviours and this consequences in the combined behaviours. These entire set of behaviours assists to outline the activities of the specific network. ABM mainly focuses on the human - social communications i.e., the process in which human being work in an organized manner and interact with each other without partaking a solitary "group mind". Straightforward discrete rules from this model results comprehensible behaviour of a specific group. Any change in the individuals within the group will create an affect the group itself for any provided populace.

Agent-based modelling is normally considered to be a experimental tool for theoretical study. ABM is a method of modelling used to understand various global patterns.

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5.7 APPLICATION IN ECONOMICS

Simulation and modelling are heavily used in the domain of economics is to study and analyse the skill, chance and strategy to simulate an aspect of reality e.g. a stock exchange". Simulation using models have become a standard in the research areas of economics. Following are the areas where modelling and simulation is used in economics.

- Simulation techniques are extensively used by economists to carry out professional researches.
- Many economic models which are dealt in higher classes of economic are structured as simulation models.
- Abstract and concrete conclusions in the domain of economics are via simulations only and this can be a beneficial aspect to the economics students. This is specifically applicable for those students who has negligible background in the area of mathematic.
- Understanding the monopolistic competition such as multiple buyers and sellers, easy entry and exit, some degree of product differentiation, zero economic profits in the long-run, etc becomes easier when appropriate modelling and simulations are used.
- Things becomes easy to comprehend using models and simulation while dealing with the controllable decisions in the area of monopolistic competition such as firm price, advertising, firm production, size of plant, etc.
- Simulation is used for teaching economic.
- Modelling enables economists to access abstract economic theories in a much easier manner.
- Models and simulations are used to develop analytical, decisive and crucial thinking abilities.
- Models and simulations support various sampling approaches in the area of statistics, mathematics and econometrics.
- Models and simulations are used in the area of business planning and subsequent economic prediction.
- Simulation of economic models are very often used in investigating various economic policy.
- Micro simulation is the process of modelling the behaviour of people and other decision components taking into consideration the associated consequences of various policy related parameters e.g., rates of tax, entitlement policies for benefits and subsidies and rates of reimbursement within the social security system. The model is simulated not only to study the effect of changes in policy on the mean behaviour but also on the total dispersal of the associated target variables. A micro simulation model consists of an array of rules, which functions on a model of micro units e.g., individuals, households and organizations.

- Predicting economic actions in a specific method where the outcome of the simulation is / are logically associated to various conventions and assumptions;
- Recommending economic policies to amend upcoming economic action.
- Offering coherent opinions to politically rationalize various economic policies at the countrywide level, to elucidate and inspire organizational policies at the level of the organization, or to deliver rational assistances for household economic conclusions at the household's level.
- Planning and allocation of fund in a country-level.
- In the area of finance, prognostic models are used for the purpose of trading (primarily in the area of investments and speculations). For example, emergent market bonds were frequently traded on the basis of economic models which are used to forecast the development of the developing countries distributing them. Additionally, long-term risk management related models have adapted the economic associations in between the simulated variables for identifying high-exposure forthcoming situations (normally accomplished via Monte Carlo method).
- Simulations and modelling are used to study the following:
  - Theories related to the choice of the consumer
  - Cost related to opportunities
  - Profits from business
  - Maximizing profits
  - Determining prices of products
  - Structure of the market
  - Demand of labours
  - Supply of labours
  - Endogenous and exogenous variables
  - Revenues distribution
  - Exploration of budgets
  - Budget constraints
  - Distortion of taxes in the real world
  - Correspondence in between general equilibrium analysis and cost-benefit analysis
  - Interconnectivities with internal and external markets
  - Tax Equivalence
  - Comparison between lumpsum and distorting Taxes
  - Optimum level of producing of public goods
  - Externalities
  - Efficiencies

Simulations in economics intends at emulating an economically relevant system by generating arrays of artificial elements and an associated recognized structure in such a method that the essentially significant attributes of the computerized model depend on the simulated relation. It is not at all necessary that the simulated relationship has to be in between the subtleties of the model and the sequential evolution of the depicted model.

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The use of simulation comprises of specific epistemological factors. For instance, a simulated system is built, the experimental activities are executed, and the yield outcomes are spotted. For instance, there is a branch of economics-influenced political science where static Monte Carlo simulations was used to learn the probability of the manifestation of the 'Condorcet paradox'. Simulation and Modelling can be categorized as generative sciences.

Following are some of the economic models and they are all simulated for research purposes.

- Cobb–Douglas production model which is used to establish the relationship between production output and production inputs. It is used in Used in economics and econometrics.
- Solow–Swan model which is used to elucidate and justify long-run economic growth. Mathematically, Solow–Swan model is considered to be a nonlinear system.
- Lucas islands model is an economic model which depicts process of linking supply of money and price. The associated output gets changed in a streamlined economy by means of rational potentials.
- Heckscher–Ohlin model states that a country should preferably export resources and elements which stands to be surplus within that country and should proportionally import items which are required by the country.
- Black–Scholes model which is also known as Black-Scholes-Merton (BSM) model is one of the most significant notions in the contemporary financial theory. This mathematical equation, provided below, yields the hypothetical value of derivatives other investment means by taking into consideration the effect of time and additional risk factors. The mathematical equation is as follows:

$$C = S_t N(d_1) - Ke^{-rt} N(d_2)$$

where:

$$d_1 = \frac{ln\frac{S_t}{K} + (r + \frac{\sigma_v^2}{2})\,t}{\sigma_s\,\sqrt{t}}$$

and

$$d_2 = d_1 - \sigma_s\,\sqrt{t}$$

where:

C = Call option price

S = Current stock (or other underlying) price

K = Strike price

r = Risk-free interest rate

t = Time to maturity

N = A normal distribution

- AD–AS model (aggregate demand– aggregate supply) portrays the determination of national income and alterations in the associated price level.
- IS–LM model - IS stands for "investment–saving" (IS) and LM stands for " Liquidity preference-money supply". It is also known as Hicks–Hansen model. It is a 2-dimensional macroeconomic model which illustrates the association between rate of interest and assets market.
- Ramsey–Cass–Koopmans model is also known as Ramsey growth model. This model is considered to be a neoclassical model of economic growth. It is considered to be one of the furthermost prevalent and extensively used macroeconomic models. In this model, the dynamics of the economy is outlined by the dynamics of capital and consumption that is seized by the system of ordinary differential equation for capital and consumption.

The Ramsey-Cass-Koopmans model, in short is also known as RCK model. It initiates with a collective production function that satisfies the Inada conditions, also known to be of Cobb-Douglas type F (K, L) with certain factors like capital (K) and labour (L). As this production function is taken into consideration as homogeneous of degree 1, it can be denoted in terms of per capita. As technology is also considered to be a significant aspect, let us consider technology (A) in the production function.

So, our production function is

$Y = F(K, AL)$

where,

Y is the output

K is the Capital with motion function $K' = Y - C - \delta K$,

A is Technology/Knowledge with motion function $A' = gA$,

L is Labour with motion function $L' = nL$

Here,

C stands for consumption

δ is considered to be the rate of depreciation of capital

g is the growth rate of technology or knowledge

n is the growth rate of labour

5.8 APPLICATION IN LIFE SCIENCE

Modelling and simulation is heavily used in association with vital mathematical environment in the area of biological science. They are used in the area of modelling uncomplicated chemical reactions to Pharmacokinetic-Pharmacodynamic Modelling (PK/PD), simulation of various biological systems, biomechanical modelling, inventing vigorous trials of drugs, and optimization of various complex parameters, analysing data, and other activities as well. More. Modelling and simulation are the irreplaceable means for scientific research in the area of biology, chemistry, biochemistry, computational chemistry, pharmacology, bioengineering and other core areas of life sciences.

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In the area of life sciences, modelling and simulations used to perform the following activities.

- Advanced researches by modelling and simulating mathematical and statistical formulas.
- Modelling and simulation enables powerful symbolic and numeric solvers for research purpose.
- Using modelling and simulation, mathematics supported logics are used are operators for rapid prototyping and progress in numerous successful solutions
- Modelling and simulation used to develop interactive applications and associated code generation for developing resolutions.
- With modelling and simulation, amalgamation of live computation and collaborative examinations with elucidations, images, graphs and others are very much feasible.
- Modelling and simulation is used for research purpose in the area of MRI technology.
- Modelling and simulation plays an imperative and essential role in generating 3-D biomechanical modelling and performing stability analysis of bipedal motion.
- Modelling and simulation is used for simulating microstructure of biomechanics of human bone.
- To generate solution capabilities in the area of Stents, Tissue Modelling, Surgical Equipment and Orthopaedics, modelling and simulation is heavily used.
- Modelling and simulation software suite consists of an extensive array of applications, templates, procedures and supports various load types for simulating human body and medical/surgical apparatus. These applications are also used to simulate to understand how the medical equipment should be used.
- Modelling and simulation techniques are used to simulate medical implanted devices (stents, artificial heart valves, dental implants), orthopaedics (artificial knees, hips), injury of brain as a result of head impact damage, tissue modelling, mechanics of feet, and portable appliances for monitoring of blood, etc.
- Modelling and simulation are used to coalesce the unsurpassed characteristics of deterministic and stochastic methods.
- Modelling and simulation are used to eliminate bottlenecks of prevailing simulation approaches.
- Modelling and simulation are used are suited to design analogous computing.

- Modelling and simulation are used in a extensive variety of simulation extents, e.g. biosciences, materials science.
- It also plays an important role in the area of public health and medicine.
- Modelling and simulation is used to study the following:
  - The heart
  - Blood Circulation
  - Exchange of gas in human lungs
  - Control of the cell volume
  - Electrical properties of cell membranes
  - The renal Counter-current mechanism
  - Mechanics related to muscle
  - Neural Systems
  - Genetics
  - Epidemics
  - Patterns of the growth of population
  - Refractory pulmonary hypertension
  - Elastohydrodynamics of vesicles in narrow, blind constrictions
  - Renal functionalities
  - Regulation of blood pressure
  - Molecular motors

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With the advent of technology, drug delivery systems and molecules with additional multifaceted architecture are generated. As a consequence, to this, the drug absorption and disposition processes, post management of the delivery of drug systems and engineered molecules have turn out to be exceptionally complicated. As the Pharmacokinetic and Pharmacodynamic (PK-PD) modelling permits for the disconnection of the drug-, carrier- and pharmacological system-specific parameters, it has been extensively referred to, to advance in understanding the in vivo behaviour of these multifaceted distribution systems and assist their improvement and expansion. This is only possible by using the modelling and simulation technique successfully. In this regard, the PK-PD modelling theory in drug delivery has been used extensively. The theory has also been used for developing new delivery systems and at the same time it is used to identify the process of modifying immense molecules. PK-PD modelling is used for developing extended-release formulations, liposomal drugs, modified proteins, and antibody-drug conjugation techniques. The model-based simulation initially uses PD models for direct and indirect PD rejoinders and to elucidate the proclamation of theoretical negligible operative concentration or threshold in the exposure-response association of various drugs and its misapprehension.

PK-PD modelling is considered to be an essential component of drug discovery and development. Its associated mathematical fashion is used to learn pharmacokinetics (PK), pharmacodynamics (PD) and their corresponding relationship.

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Figure 5.5 illustrates, the mechanism-based PK-PD model which can be combined into various phases of development of drug. Unambiguously, PK modelling is used to numerically portray the process of absorption and disposition of drug within a human body. PD modelling assesses the time course of the pharmacological consequences of drugs, with the contemplation of the method of action of drug and foremost rate-limiting stages within the biology of a human system.

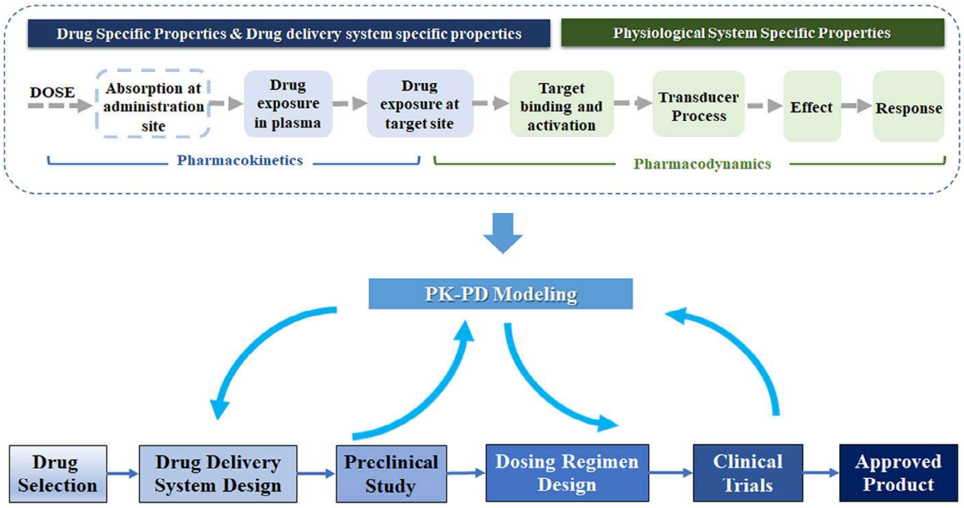

Fig. 5.5 *Mechanism-Based PK-PD*

The PK and PD modelling can enumerate the association of drug exposure and response, and supplementary, depict the impacts of drug-specific, system-specific delivery, system-specific parameters related to physiological and pathological and its corresponding affiliation. Parameters which are drug-specific (e.g., drug clearance and receptor binding affinity) exemplify the communication and collaboration amid the drug and the corresponding biological system. The drug delivery system-specific parameters signify the attributes of transporters, such as the clearance, release rate and the internalization rate of the transporter. The physiological system-specific parameters characterize physiological entities such as blood flow, life-span of cells, expression of enzymes, and transporters.

Mechanism-based PK-PD modelling is used to enumerate the Minimum Effective Concentration (MEC) of Erythropoiesis-Stimulating Agents (ESAs), and elucidate the paradox that ESAs with lesser binding affinity has a higher in vivo action.

Let us consider another example.

An organization has developed a model of a living human heart human. It is of high-level trustworthiness and is a multiphysics model of a healthy human being. It consists of 4-chamber adult human heart and proximal vasculature. The response of the heart model is administered by accurate electrical, structural, and fluid flow physics. The model encompasses of a ready-to-execute lively, electro-mechanical simulation, sophisticated geometry, a blood flow model and a comprehensive depiction of cardiac tissues comprising of inactive and active behaviours, its fibre-like character and the associated electrical alleyways.

The simulated living heart is used in learning cardiac imperfections and unhealthy conditions and discover decisions related to treatment. Model characteristics can be altered or re characterised exclusively along with the associated geometry, settings related to loads and boundary and material related attributes. Medical appliances can be inserted into the model to find out or study their effects on cardiac functionalities, to authenticate their effectiveness and to forecast their consistency underneath a extensive variety of operational situations.

Case Study

Maple and MapleSim plays a very important part in 3-D biomechanical modelling and stability analysis of bipedal motion.

Biomechanics is a branch of science which is associated with the interior and exterior forces acting on the human body and the associated consequences initiated by these applied forces. A big portion of this field today encompasses of using software applications to establish a model which is precise to human biomechanical movements e.g., running, walking, jumping, and standing. MapleSim is a software which is used for physical modelling and multidomain simulation. In this situation this software is used.

The Control Systems Lab researchers at the University of Arkansas at Little Rock is responsible for conducting research in this area of study.

Sit-to-stand is a normal task of a human being. It consists of the amalgamation of a musculoskeletal structure combined with neural control. Along with this it consists of innumerable number of variables. The simulated movement is examined for an 8-segment inflexible body model with physiological motor operations.

The real mechanism which is to be modelled is extremely multifaceted with a system in 13 degrees-of-freedom and 7 linkages with sagittal and frontal angles. To accomplish the anticipated motion, the scientists required to identify the applicable associated torques and moments without disrupting the holonomic constraints in the model within the complete manoeuvre. In addition to this, the scientists required to perform a thorough analysis about the constancy and consistency of the system by means of countless conditions and norms, as well as Lyapunov's Indirect Method.

The model is given inFigure 5.6 for reference.

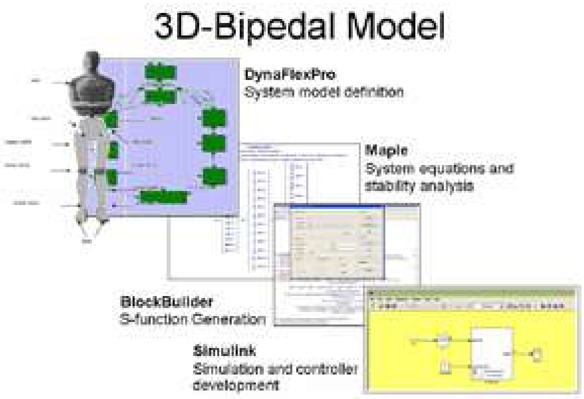

Fig. 5.6 *3D Bipedal Model*

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Using MapleSim application, the scientists were successful in producing the bipedal model to learn the sit-to-stand transfer manoeuvre. By choosing the joints, bodies and frames cautiously, it was uncomplicated for them to establish the biomechanical models successfully. Manually obtaining the analytical equations for a high-order system is almost next to impossible.

The associated equations related to analytical math were swiftly demarcated from the model without additional coding. Once the equations were comprehensive, they were transported into Maple application for performing stability analysis.

The consequences of this modelling progression delivered an improved interpretation of the human physiological system at the time of sit-to-stand movement. The improvements at various joints demonstrates points where augmented exertion is required within the movement. This process also demonstrations the correlation among diverse kinematic variables and its associated consequence on augmenting the voluntary movement.

5.9 APPLICATION IN DATABASE DESIGN

Modelling & Simulation is highly used to design a database. The objective of designing a database is to bring data representation and its associated relationship for analysis and testing functions. The notion of data model was first announced in 1980 by Edgar Frank "Ted" Codd. Following are the prominent characteristics of the model.

- Database is the assortment of various data objects that describes the information and their relationships.
- Rules are used for outlining the constraints on data in the objects.
- Operations are applied to the specified objects for extracting information.

Earlier, Data Modelling was on the basis of the perception of entities & relationships where the entities are considered to be the types of information of data, and its relationships characterizes the relations between the entities.

The latest concept for data modelling is based on object-oriented design where entities are characterized as classes, which are normally used as templates in during coding programs. A class consists of its name, attributes, constraints, and relationships with objects of other classes.

Its basic depiction Figure 5.7 is as follows:

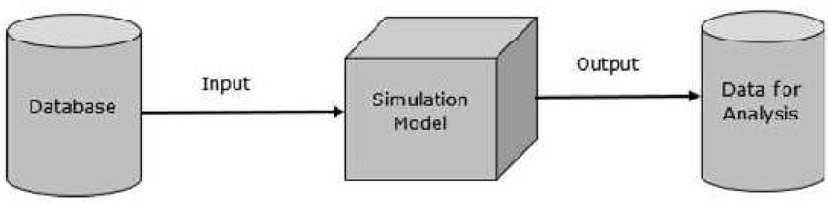

Fig. 5.7 *Database Simulation*

Data Representation

- Data Representation for Events

  A simulation event consists of attributes i.e., the name of the event and its accompanying time information. It characterizes the execution of a given simulation which uses a a set of input data connected with the input file parameter and generates its consequence or results as a set of output data which is stored in multiple files connected with data files.

- Data Representation for Input Files

  Every simulation process necessitates a dissimilar or various set of input data and its accompanying parameter values, which are embodied in the input data file. The input file is related with the software which is responsible for processing the simulation. The data model characterizes the referenced files by an connotation with a data file.

- Data Representation for Output Files

  When the simulation process is accomplished, it generates numerous output files. All output files are generated in the form of a data file. Each file has its corresponding unique name, explanation and a universal factor. A data file is categorised into two files.

  - The first file consists of numerical values
  - The second file consists of expressive information about the contents of the first file (numerical file).

Typical Database Design Life Cycle

Database design is complex and the level of complexity is dependent on the complexity of the associated project. As far as the authentic stages in consecutive order goes, there are multiple stages that a database system should go through at the coding phase.

For reduced database systems, the development life cycle is naturally fairly easy, with few stages to go through. The stages should be as follows:

- **Requirement analysis**: In this stage the mapping of the development life cycle is done. During this stage, the scope and limits of the database system are also well-defined.
- **Database designing**: In this stage logical and physical designing models are prepared. The logical model is normally accomplished using pen and paper, without any physical deployment or any specific DBMS contemplations. The physical model then establishes the logical model, while considering identified other enactments into consideration.
- **Implementation**: In this phase, the data is converted and loading into the database. This stage consists of importing and converting data from the former/outdated system into the new database, along with the testing phases.

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Database development life cycle

Figure 5.8 illustrates the database development life cycle

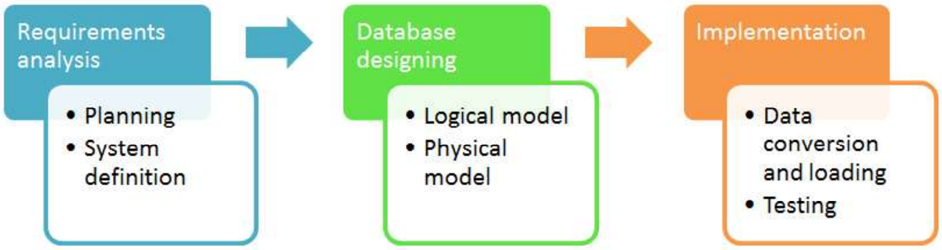

Fig. 5.8 *Database Development Life Cycle*

Database development life cycle - The database development life cycle has a various phase which are maintained during the development of database systems.

It is not necessary that the steps need to be followed in a sequential manner.

For small database systems, the course of database design is very simple and do not require to follow the complex steps.

To completely understand the above illustration, let us take a deep dive at the various components enumerated in every step for understanding the DBMS design process.

Requirement Analysis

- **Planning** – These stages of the database design concepts are associated with planning of complete Database Development Life Cycle. This phase considers the strategy of Information Systems of the company.
- **System definition** – The scope and boundaries of the planned database system are well-defined in this phase.

Database Designing

- **Logical model** – This stage is about developing a database model based on planned necessities. The complete design is done with pen and paper without any physical applications or specific DBMS deliberations.
- **Physical model** – This stage is concerned about the implementation of the logical model of the database taking into consideration the DBMS and physical deployment aspects.

Implementation

- **Data conversion and loading** – This is the stage which is concerned with the importing and converting data from the legacy system into the newly designed database.
- **Testing** – The errors are being identified in the currently implemented system. This phase compares the database against requirement specifications.

Following picture depicts an Entity-relationship diagram where we can see relations are established between the data elements.

Entity Relationship Model (ER Modelling) is a graphic method to database design. It is a high-level data model that describes data elements and their corresponding relationship for a specific system. An ER model always represents objects from the real-world (Fig. 5.9).

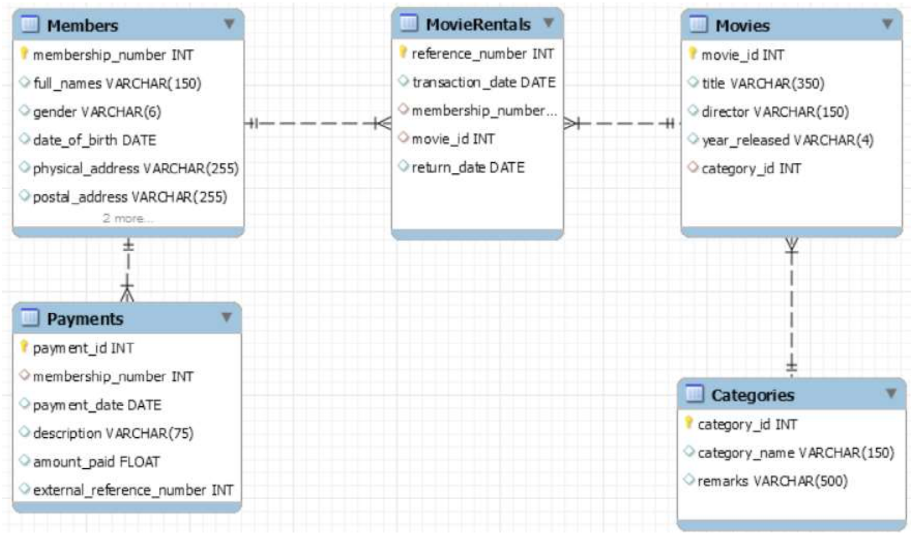

Fig. 5.9 *E-R Diagram*

Feature of an efficient database design are as follows:

- Splits information into subject-based tables so that redundant data eliminated.
- Delivers Access with the information as required appropriately to link the information in the tables together as required.
- Assists in supporting and ensuring the correctness and integrity of the information.
- Accommodates the processing of data along with reporting requirements.

The design process consists of the following activities and they are the most important once during creating the model database:

- Regulate the purpose of the database
- Discover and systematise the information required
- Split information into various tables
- Convert information items into columns
- Derive the primary key
- Establish relationships among the tables in a database
- Improve and enhance the database design
- Applying the normalization rules as appropriate

A well-defined database shall

- Save disk space by removing redundant data.

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- Maintains the data accurateness and integrity.
- Provides access to the data as required basis.

To avoid these complex steps, database is modelled and simulated to check the correctness and its behaviour during processing data.

5.10 APPLICATION IN COMPUTER DESIGN

Modelling and simulation is a recognized scientific and business technique to backing engineers and scientists in their activities in all the possible phases of a lifecycle, i.e., from tendering to operation activities and catered service for a technical system. As intricacies of such information technology associated systems are increasing in the area of ERP, hardware and infrastructure related systems. In such case, to make the design effective, modelling and simulation is frequently used. Simulation is also used to scrutinize and evaluate the client-server systems and provide assistance in effective and efficient feasible deployment of client-server related solutions. Client–server architecture is a distributed architecture which panels processes or capabilities in terms of workload between the service providers known as servers and associated requester of services known as clients.

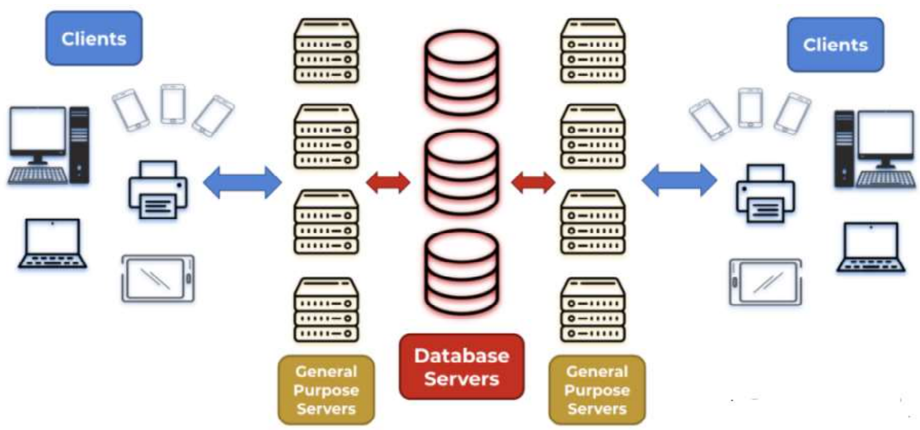

Fig. 5.10 *Client-Server Architecture*

Simulation is modelling is used extensively in studying and analysing in the following areas:

- Monitor the data flow within a wide area network
- Component designing in client-server architecture
- Monitoring load balancer
- Protocols
- Throughput monitoring and analysing
- Failure points of a wide area network
- Cashing behaviours, including Cache Invalidation, Write-through cache, Write-back cache, Cache Aside, Read-Through Cache, etc

- Monitoring of Cache eviction policies e.g., First In First Out (FIFO), Last In First Out (LIFO), Least Recently Used (LRU), Least Frequently Used (LFU), Random Selection, etc
- Designing of rate limiters
- Memory estimation of super computers
- Parallel processing
- Latency calculation
- I/O monitoring in a database
- File transfer efficiently
- Data Synchronization
- Processing speed and capabilities of a processor
- Monitoring of server scalabilities
- Study of Neural Network
- Artificial Intelligence
- Supervised and unsupervised Machine learning
- Threat management
- Risk management
- Detection of exposure to complex threats and vulnerabilities
- Data normalization techniques
- Data compression ration calculation
- Aggregation of events
- Business Intelligence
- Analytics
- Security orchestration

Using simulation to assess the computer and network design is not at all very straight forward. Simulation strategies for problem-solving certainly necessitates an excellent intellect of simulation modelling and the associated computation and statistical characteristics of simulation. For example, to understand the complexity of client-server model we can refer to the following statement.

"A computing model whereby the application processing is partitioned across multiple processing platforms and there processors cooperate to complete the processing as a single integrated task" - IES Committee (IESC) set up by the Australian Federal Department of Finance (1995).

According to Gartner Group (1994), the client-server consists of five configurations via the blending of the three components. They are:

- Presentation
- Processing
- Data management

These three components are distributed across the client(s) and the server(s) in five various conducts. They are as follows:

-Distribution presentation

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- Remote Presentation
- Distribution Function
- Remote Data Access
- Distributed Database

Figure 5.11 depicts the five different components

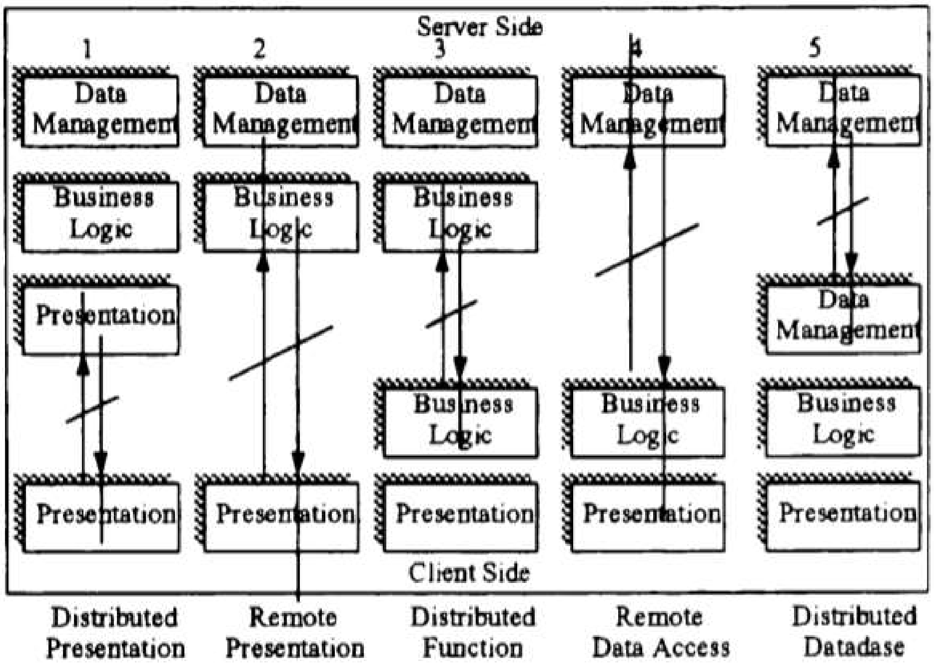

Fig. 5.11 *The Gartner Group Models of Client/Server Systems*

Performance of such system will certainly depend on various aspects e.g., the network topology, protocols used for transmission, processor speed of client systems and servers, number of supported clients, choice of operating systems and other applications (which may be resource hungry) and associated network traffic.

So, to put this architecture into reality and considering all the said factors, simulation and modelling is extensively used.

Let us also examine the complexity of data exchange from and to clients provided in the below figure.

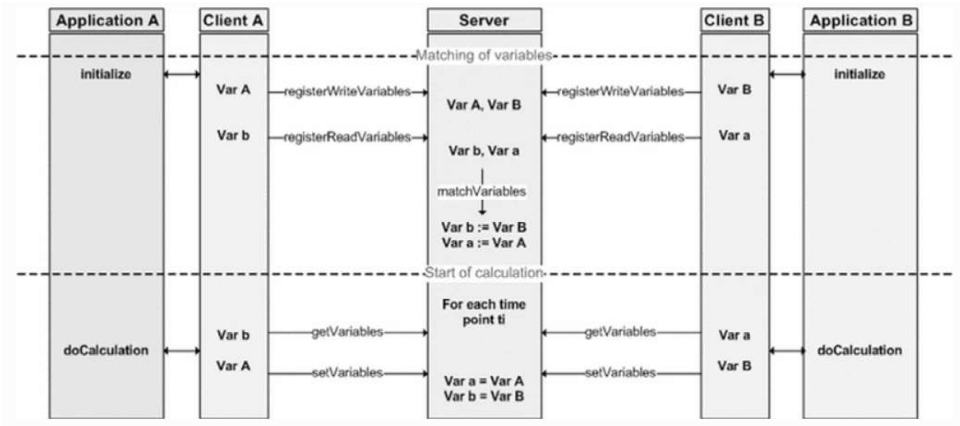

Fig. 5.12 *Data Exchange from and to Clients*

Figure 5.12 depicts that the server is deployed with Application A and Application B with various versions i.e., Ver A and Ver B. Client A and Client B is accessing Application A and Application B respectively. During application, caching, memory read/write and hard disk read/write related activities are taking place. This picture seems to be simple with just two users but to replicating this in a real life where millions and millions of users are accessing these applications (may be numerous) a rigorous simulation is a mandatory to learn each and every behaviour of the application else the entire system will collapse.

Following are some of the examples of software failure which created a huge ripple in terms of business, globally:

- Heathrow disruption - February 2020
- British Airways - 2019
- Facebook, Instagram and WhatsApp - All disrupted for 14 hours on July 2019
- O2 service disruption December 2018
- WannaCry attack May 2017
- Memory Failure - Tesla Recalls 135,000 Vehicles
- Cisco's Email Security Appliances glitch - 2016
- Deadly Flaw in Medical Infusion Pumps - 2015
- HSBC Online Banking Failure- April 2016
- Bitcoin Node Crash – April 2017

Further investigation proved that all these incidents took place as associated software was not tested in a proper manner. In such case to do a thorough testing from all aspects, simulations help a lot to the software developers and it is a mandatory.

Let us consider another complex situation of client scheduling. Figure 5.13 depicts a complex set of activities during job execution.

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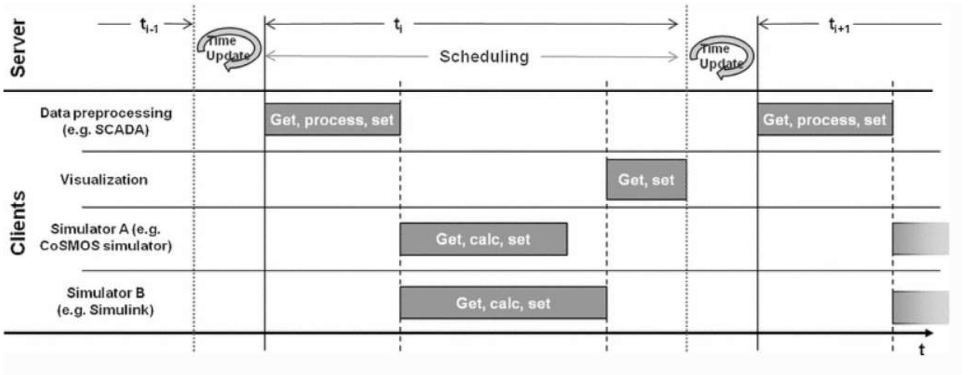

Fig. 5.13 *Client Job Scheduling*

In this picture we can see the complex process involved within a system when jobs (scheduled and unscheduled) get processed or executed. The above picture provides an illustration during a simulation execution during job processing for multiple clients. In such case, a simulation is a mandatory and this is used by the engineers and scientists to design optimal job management solution for an operating system. Once simulated, the simulator provides optimal value using which the operating system can be design for handling jobs.

Now let us consider another complex situation in relation to the real-time clocked co-simulation as per Figure 5.14.

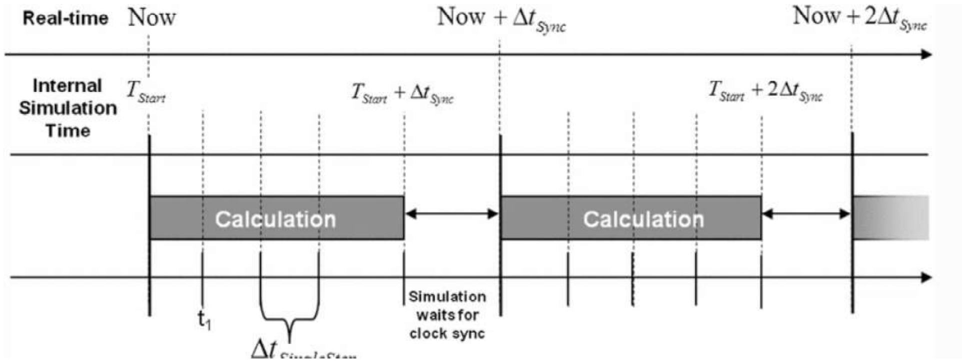

Fig. 5.14 *Real-Time Clocked Co-Simulation*

This perception describes each simulation run as a independent job. One job comprises of a comprehensive client–server setup.

The job notion inserts to the configuration of a independent job run i.e., involving the client setup and associated configurations along with certain fundamental job run related settings i.e., an widespread configuration related to time control. Each job is normally programmed with a initiation time in real time and synchronized with real-time at quantified and definite synchronization time points. Additionally, the job can certainly get associate with its time behaviour to real-time labels (which is the virtual real-time).

Simulation steps are executed intermittently in the real-time situation of a specific operational support system.

To appropriately control the jobs, a job manager (internal software component) is deployed. Apart from handling an independent sole job, the job manager can handle multiple jobs as well as configuring it accordingly. If the number of processing components permits, specific jobs which overlaps during its execution time interval will certainly be executed parallelly on various processors. In case the availability of free processor is zero or the processor is not able to execute specific jobs in parallel, the jobs get executed as per its predefined priority. A job having lesser priority is interrupted for future resumption and instead a job having higher priority is executed. This is where simulation plays a very important role. The entire scenario is modelled within the simulator to observe the job scheduling process in more complex situations.

The sequence control of the simulator deal with event-discrete, time-discrete and continuous models parallelly and additionally permits Differential-Algebraic Equations (DAE) in the continuous modelling, which are resolved in one comprehensive equation system overall by means of distinct, time efficient DAE-resolvers for the specific networked systems in a scientific manner. This consents the resolution of discretized partial differential equations also. The sequence control maintains a usual hierarchy, based on the principle of the automation pyramid which is portrayed Figure 5.15. The time discrete evaluation intrudes the unceasing incorporation for exchange of information. Events spawns a precise assessment in the time discrete and succeeding in the continuous model.

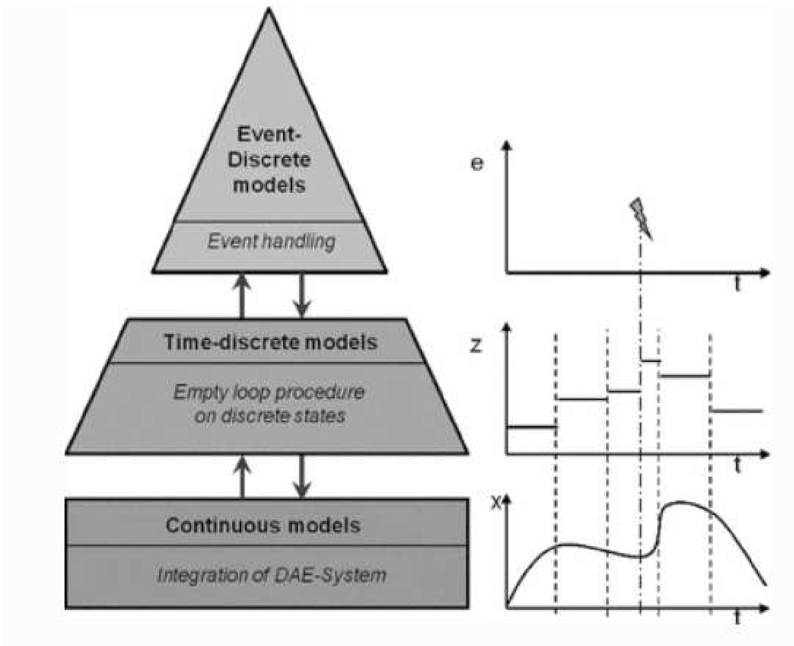

Fig. 5.15 *Three Different Models and Their Relation*

In time and event discrete modelling, normally, specific states are demarcated they are assessed either every fixed discrete sample rate or whenever any event take place. So, automation and control logic conduct is characteristically modelled time discrete and event-discrete modelling is used primarily in event-driven domains e.g., logistics traffic and transport and related systems.

Fuzzy sets are also used in modelling and simulation extensively.

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Use Cases

Conducting an Operational Research via Simulation

Business professionals normally uses operations research related methodologies like analysing data to create improved choices for associated business. They use an array of designated data to scrutinize multifaceted circumstances which in turn returns predictions concerning about the associated risk factors in a cost-efficient fashion. Simulations enables to bring down various options and perform respective mining of data to identify beneficial acquaintances. Use your conclusions to make a more reliable forecast. Conducting operational research involves gathering data, analysing data and drawing conclusions. The steps for this activity is as follows:

- Relevant data is collected for the purpose. Operations research related techniques are used to design a computer network, flow of the materials is managed and decided, relevant processes are automated, scheduling of systems are optimized, optimal prices are determined. For example, information about all support cases of client for a specific item within the organization are collected. Normally, support case related information consists of demographic data related to the customer, associated procurements, various symptoms of the incident, actions related to resolving the incident and resolution related steps & other information.
- With the help of operations research methodologies, the data needs to be scrutinized. This step will assist to take decisions which are based on substitutes, limitations and assessment benchmarking. Data can be scrutinized with the help of data mining, probability and statistics related methodologies. Data mining is about organizing or assembling the data into various clusters and then categorizing them with specific characteristics. For example, while performing an analysis of server logs of a specific client, the issue with the server can be understood. Along with this information related to the server's serial number can be extracted. From this manufacturer's location can also be traced. Similarly, while scrutinizing the procurement history of a customer, product details can be extracted. This kind of analysis is normally used to effect procurement related decisions.
- In case analysed simulated data is existing in internet, they can also be referred to for understanding the trends. For instance, the NOVA On-line fire-growth simulation demonstrates the effect of wind speed and direction influencing the spread out of fire. At the same time, it suggests how the fire can be controlled by the firefighters. Business simulations also helps in changing the input values to understand the effect of the alternation. This kind of exercise is used to observe the cost controlling procedure. For example, "Consortium for Service Innovation" provides a spreadsheet which can be used to calculate sample data. For saving time and money, these kinds of spreadsheets can be used to simulate data and implement the outcomes or resultants. This will also be cost effective.
- Post analysis the research should be concluded. To provide a meaningful presentation for higher management, the resultant data should be summarized and converted into a presentable format. The presentation should also contain

the challenges and the mitigation procedures. The presentable format of the report will be senior management or the leadership team to take further decisions. To qualify the research successfully, source and time period should also be defined in the presentation. The research presentation should also consist of the required logs as well along with the time period of the logs. A trend should also be in the presentation as appropriate. Proper solutions should be provided in the presentation For example, to cater better service by a laptop support centre, additional employees should be employed during the morning shift for better support to the customers.

Defining Budget for Developing Software

Crafting an effectual budget for software development normally depends on complexity of the developing, technology used and hourly cost of the engineers along with the number of days required for the development. It also consists of models for estimating which will be used for development. To find out the project cost related to software development in an efficient manner, the first and foremost thing is to identify the scope and complexity of the development. Time is also another factor which needs to be taken into consideration. If the development is complex with huge number of features, then the development will take more time. Hence the requirements are to be listed, manhour needs to be calculated which once put in the simulator will generate the budget accordingly. A proper budgeting and planning will help in delivering the software development successfully and on time.

- An overall estimation for cost and schedule needs to be created via a "model-based, expertise-based, learning-oriented or dynamics-based" method. On the basis of the kind of project, output of the model-based methodologies can be selected, keeping in mind the Software Development Life Cycle (SDLC) stages for estimating the project effort, timelines and associated rate of defects. Supplementary significant model dimensions comprise of hours of development, number of developers required for development and testing, associated risks, costs, hardware needed and impact of the portfolio. There are certain expertise methodologies like Delphi which is used for questioning methods. Project managers should collect all the required data to build the development models which will assists in addressing shortfalls. Dynamics-based methodologies are used to identify the deadlines, level of resource, various requirements for the project, appropriate training and additional factors which are oscillating in nature during software development related activities.
- A risk analysis in this regard is a mandatory conducted for identifying the factors which can create a negative impact over the project. Factors can be the size of the system, project management related reporting and technology related glitches. Once the risk factors are identified, corresponding mitigating actions can be plotted out so that the project finishes within time, smoothly. Some examples of this factor are the costs for scope related decisions, project stakeholders and allocations of hardware.

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- Project should be planned with standard project management tools. This tool should be used to track the progress of the project. In case of any delay identified, appropriate actions can be taken.
- Future improvements with respect to the developed software has to be identified via thorough analysis. The aim should be towards product maturity.

Budgeting Tools for Project Management

Budgeting tools and techniques are used by project managers to formulate finance related data which assures that a project is sponsored sufficiently and can be accomplished within the approved budget. Project managers accomplishes these finance related activities and calculations using appropriate models, various templates and calculating means and other means. These tools are used to evaluate the calculations precisely. At the same time these tools are also capable enough to do the project monitoring prudently along with features related to risk management. These tools are used by the project managers to efficiently correspond the status of the allocated budget to the project sponsor. Project sponsor is responsible for allocating project budget.

- **Standards**

  Budgeting a project consists of the following steps.

  - Break down your project into tasks and milestones : Project scope and the task list is to be created with specific timelines
  - Estimate each item in the task list : All required resources and materials requires to be identified and their estimation should be included while calculating the project cost.
  - Add your estimates together : After identifying the tasks and costs, the total cost is to be calculated.
  - Add contingency and taxes : Contingency has to be planned in this phase so that the project should not get hampered for lower or no budgets.
  - Get approval : Once the total cost of the project is calculated, the project sponsor should approve it.

- **Analogous**

  This tool works mainly with the actual costs of a preceding project to find out the cost of a current project. The condition here is this that both the project has to be of similar types. Normally this methodology is used by project managers who are new to project management. This approach is also applicable for small and less complex projects. For complex projects this approach may not be appropriate.

- **Parametric**

  This tool works mainly with the erstwhile project data and other associated variables for estimating of the project parameters, e.g., project scope, cost involved in the project and time duration of the project.

- **Top-Down Method**

  With this budgeting method, the whole project is looked down upon and then the budgeting and required estimation for each process is accomplished.

By scrutinizing every task required to conclude the project, the assessment of prioritizing the tasks can be done. The time limit is also estimated from this as per the allocated budget. This approach does not support the "participatory-style management". In Top-down estimation approach, the process in gist is to identify or calculate the total activities and then fragment the main activity it into sub-tasks or milestones as appropriate.

- **Bottom-Up Method**

  It is just the opposite of top-down approach. Here, the individual parts of the project plan are rated and then they are summed together. This approach is one of the proficient and best approach for preparing a project budget. It considers all the discrete portions of the project, e.g., project tasks, project milestones, and they sums them to achieve the cost involved in a project. This method is very much applicable during the point of creatin of a declaration of the work. In case all the components of the project is known, then the bottom-up approach is the best way to move ahead.

Features of a Software Simulation Tool

- Modelling feature :Interaction with various processes, event standpoints and incessant modelling, requirement dependable
- Competence in analysing Input-data: Should be capable to mathematically or statistically estimate the data distributions from the raw data.
- Model-building, graphically: It consists of various process flows, block-diagram and network diagrams. For database designs, ERD (Entity relationship diagram) is also supported
- Routing of entities with conditions: Supports the routing of entities on the basis of predefined conditions or appropriate associated attributes
- Simulation programming : Procedural logics can be appended using high-level powerful simulation language
- Flexibility of input data : It is capable of accepting data from various sources i.e., external files, databases, spreadsheets, or other sources
- Conciseness of modelling : Supports various actions, block or nodes
- Randomness : Able to generate Random-variate generators for every type of distributions, e.g., Exponential, Triangular, Uniform, Normal
- Specialized components and templates: Consist of capabilities like handling of materials, vehicles, conveyors, bridge cranes, handling of liquids and bulk materials, communication systems, call centres, etc.
- Consists of user-built custom objects templates and sub-models
- Interfacing is possible with C, C++, Java or other programming language
- Simulation-enabled designs can be performed for manufacturing and engineering of products
- This kind of software can perform "progressive nonlinear analysis" and "spot weld fatigue analysis" to accomplish additional intensifying the speed and memory effectiveness.

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- Powered by artificial intelligence, this software makes use of machine learning to generate condensed order models of prolonged runtime, extremely complex subsystems models
- Available readymade solutions in this kind of software include structural, thermal, flow, electromagnetics, injection moulding and welding related engineering activities.
- Using this software, specialized applications, e.g., V2X communications can certainly be simulated with supplementary correctness using the "Shooting and Bouncing Rays (SBR)" methodology.
- Simulation can be accomplished to identify certain attributes of a material like rigidity, strength, constancy, electrical and analysis of thermal transfer.

Check Your Progress

10. What is simulation and modelling in economics.
11. Write the use of modelling and simulation in life science.
12. What is data base design?
13. Define the term cliend and server.

5.11 ANSWERS TO 'CHECK YOUR PROGRESS'

1. Simulated environments permit and assists us to assess innovative thoughts and concepts prior to making complex and multifaceted decisions. This analysis technique allows us to deploy various parameters within a system, to identify numerous prospects for development and enhancement of the current system.
2. Simulated environments are used simulate new ideas prior to assembly a multifaceted business verdict. This kind of analysis procedure permits you to deploy various parameters, for instance income & expenditures, to determine various prospects for improving in the ongoing operational activities, etc.
3. Simulation optimization is defined as the procedure to identify the unsurpassed input variables from all likelihoods of data sets without specifically assessing every single data. The primary purpose of simulation optimization is to curtail down the resources expended while make best use of data attained in a simulation trial.
4. Optimization primarily is dealt extensively in two main areas of Operations Research (OR) are:
   - Optimization parametric (static)
   - Optimization control (dynamic)
5. Heuristic method alters accurateness by speediness. The main aim is to identify a faster method for solving a problem. Normally this process identifies the local optimal value as an alternative to the optimal value. The values are

very near to the concluding solution. Tabu search and genetic algorithms are some of the examples for this method.

6. Artificial intelligence known as AI in short is a vast area of study in computer science which deals with creating smart computers which are proficient in accomplishing activities which normally necessitates intelligence of human being.
7. Scientists like Norvig and Russell defined AT as:
   - Thinking like human being
   - Thinking sensibly and realistically
   - Acting like human being
   - Acting sensibly and realistically
8. Examples of artificial intelligence are:
   - Google uses AI during searching
   - Software used for image recognition
   - Vehicles supporting auto-driving
   - Watson from IBM
   - Siri, Alexa and other smart applications related to personal assistance
   - Robo-advisors
   - Typical bots
   - Used to filter spams in emails
   - Recommendations by Netflix
   - Google's AI-powered forecasting (e.g., Google Maps)
   - Applications related to rides (E.g.: Uber cab, Ola cab, etc)
9. Natural Language Processing (NLP) is the capability of computers to analyse, comprehend and produce human language which includes speech as well. The subsequent stage of NLP is natural language interaction, which permits humans to interact with computers by means of usual, day to day linguistic to accomplish certain tasks.
10. Simulation and modelling are heavily used in the domain of economics is to study and analyse the skill, chance and strategy to simulate an aspect of reality e.g. a stock exchange". Simulation using models have become a standard in the research areas of economics.
11. Modelling and simulation are used in the area of modelling uncomplicated chemical reactions to Pharmacokinetic-Pharmacodynamic Modelling (PK/PD) modelling, simulation of various biological systems, biomechanical modelling, inventing vigorous trials of drugs, optimization of various complex parameters, analysing data, and other activities as well. More. Modelling and simulation are the irreplaceable means for scientific research in the area of biology, chemistry, biochemistry, computational chemistry, pharmacology, bioengineering and other core areas of life sciences.

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12. Modelling & Simulation is highly used to design a database. The objective of designing a database is to bring data representation and its associated relationship for analysis and testing functions. The notion of data model was first announced in 1980 by Edgar Frank "Ted" Codd.
13. Client–server architecture is a distributed architecture which panels processes or capabilities in terms of workload between the service providers known as servers and associated requester of services known as clients.

5.12 SUMMARY

- Simulation models provide us graphical representation of information which can be certainly be edited and animated, portraying us the consequences of grasping specific actions.
- Simulated environments are used to simulate new ideas prior to assembly a multifaceted business verdict. This kind of analysis procedure permits you to deploy various parameters, for instance income & expenditures, to determine various prospects for improving in the ongoing operational activities, etc.
- Simulation models can provide us an explicit presentation or demonstration of statistics that can be amended and animated, displaying us the consequences in case if we adapt specific activities and operational actions.
- Simulation optimization is defined as the procedure to identify the unsurpassed input variables from all likelihoods of data sets without specifically assessing every single data. The primary purpose of simulation optimization is to curtail down the resources expended while make best use of data attained in a simulation trial.
- Response surface methodology is to identify the association in between the input variables and the response variables. The process initiates with attempting to adapt a linear regression model.
- Artificial intelligence known as AI in short is a vast area of study in computer science which deals with creating smart computers which are proficient in accomplishing activities which normally necessitates intelligence of human being.
- Patrick Winston, who was the "Ford professor of artificial intelligence and computer science" at MIT, has described artificial intelligence as "algorithms enabled by constraints, exposed by representations that support models targeted at loops that tie thinking, perception and action together."
- Machine learning mechanizes or automates the constructing of the analytical model. This method uses approaches from the field of neural networks, statistics, operations research and physics to identify the veiled understandings and acumens in the array of data without evidently being programmed for what needs to be done or where to find or what to find.
- A neural network is a category of machine learning process which comprises of interrelated or interconnected or unified components or units (e.g., neurons)

which processes information by replying to exterior inputs, depending data between each component or unit. The process needs various passes at the data level to identify associated acquaintances and generate sense from the undefined array of data.

- Deep learning uses enormous neural networks with various layers of processing units, captivating benefit of advancements in the power of computing and enhanced training methods to study multifaceted patterns in enormous amounts of data. Usual applications consist of recognition of images and speech.
- Sociology is the area of human social relationships and organizations.
- Simulation and modelling are heavily used in the domain of economics is to study and analyse the skill, chance and strategy to simulate an aspect of reality e.g. a stock exchange". Simulation using models have become a standard in the research areas of economics.
- Modelling and simulation is heavily used in association with vital mathematical environment in the area of biological science.
- Modelling & Simulation is highly used to design a database. The objective of designing a database is to bring data representation and its associated relationship for analysis and testing functions. The notion of data model was first announced in 1980 by Edgar Frank "Ted" Codd. Following are the prominent characteristics of the model.
- Modelling and simulation is a recognized scientific and business technique to backing engineers and scientists in their activities in all the possible phases of a lifecycle, i.e., from tendering to operation activities and catered service for a technical system. As intricacies of such information technology associated systems are increasing in the area of ERP, hardware and infrastructure related systems. In such case, to make the design effective, modelling and simulation is frequently used.

5.13 KEY TERMS

- **Simulation Optimization:** Simulation models can provide us an explicit presentation or demonstration of statistics that can be amended and animated, displaying us the consequences in case if we adapt specific activities and operational actions.
- **Optimization Parametric (Static)**: The main intend is to identify the values of the parameters, which are "static" in nature for all possible conditions, with the objective of increasing or decreasing a functionality. For this, linear programming is used. Simulation for this assists a lot, in case, noises are existing in the parameters or assessment of the difficulty would request for tremendous computing time, as the situation is very complex in nature.
- **Optimization Control (Dynamic)**: This kind of optimization is only used in the area of computer science and electrical engineering. The optimum regulator is per state basis and the outcome gets altered in each of the

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states. Normally mathematical programming or dynamic programming is used in this area. Simulation produces random samples and resolves compound and large-scale issues.

- **Artificial Intelligence:** Artificial intelligence known as AI in short is a vast area of study in computer science which deals with creating smart computers which are proficient in accomplishing activities which normally necessitates intelligence of human being.
- **Natural Language Processing (NLP):** It is the capability of computers to analyse, comprehend and produce human language which includes speech as well. The subsequent stage of NLP is natural language interaction, which permits humans to interact with computers by means of usual, day to day linguistic to accomplish certain tasks.
- **System Level Modelling (SLM):** System Level Modelling (SLM) targets to precisely forecast and express or suggest various amount of actions, behaviours or other hypothetical options of person, object, construct, etc., within a specific system by means of a huge array of mathematical equations and computing power in form of various appropriate models.

5.14 SELF-ASSESSMENT QUESTIONS AND EXERCISES

Short-Answer Questions

1. Write the function of simulation model.
2. What is the application of simulation and modelling in management.
3. What is simulation optimization?
4. State the artificial intelligence.
5. Write the definition of NLP.
6. Learn about the application of simulation and modelling in economics.
7. How will you define the application of simulation and modelling in life science?
8. What do you understand by the typical database design life cycle?
9. State the application of simulation and modelling in computer design.

Long-Answer Questions

1. Discuss briefly about the simulation model with the help of examples.
2. Explain the application of simulation in management. Give appropriate examples.
3. Briefly discuss about the simulation optimization with the help of examples.
4. Describe the applications in artificial intelligence with the help of relevant examples.
5. Analysis the application of simulation and modelling in sociology. Give appropriate examples.

6. Discuss the various application of simulation and modelling in economics with the help of relevant examples.
7. Elaborate on the application in life science with the help of giving examples.
8. Explain the database design. Give appropriate examples.
9. Discuss applications in computer design with the help of examples.

5.15 FURTHER READING

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